Policy invariance under reward transformations: Theory and application to reward shaping

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(presented by Toryn Klassen)

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Outline

MDP review

Reward shaping

To provide guidance, policies can be learned on an MDP with a modified reward function, and then used on the original MDP (with varying results).

Potential-based reward shaping

To ensure that good policies for a modified reward function are also good for the original, it suffices to base the rewards on a **potential function**.

Experiments

Some potential-based shaping functions are evaluated.

MDP review

Definition

A **Markov decision process (MDP)** is a tuple $M = \langle S, A, T, \gamma, R \rangle$ where

- S is a finite set of **states**,
- $A = \{a_1, \ldots, a_k\}$ is a set of **actions**,
- $T = \{P_{sa} : s \in S, a \in A\}$ specifies **transition probabilities**; $P_{sa}(s')$ is the probability of transitioning from s to s' with action a,
- γ is the **discount factor**, and
- $R : S \times A \times S \to \mathbb{R}$ is the **reward function**.

Definition

A **policy** over a set of states S is a function $\pi : S \to A$.

MDP review

Definition

Given a policy π and MDP $M = \langle S, A, T, \gamma, R \rangle$, the **value function** V_M^π is defined by

$$V_M^\pi(s) = \mathbb{E}[R_1 + \gamma R_2 + \gamma^2 R_3 + \dots ; \pi, s]$$

where R_i is the reward received on the ith step of following π, starting from s.

Definition

The Q-**function** is

$$Q_M^\pi(s, a) = \mathbb{E}_{s' \sim P_{sa}}[R(s, a, s') + \gamma V_M^\pi(s')]$$

MDP review

- The **optimal value function** is $V_M^*(s) = \sup_\pi V_M^\pi(s)$.
- The **optimal Q-function** is $Q_M^*(s, a) = \sup_\pi Q_M^\pi(s, a)$.
- The **optimal policy** is $\pi_M^*(s) = \operatorname{argmax}_{a \in A} Q_M^*(s, a)$.

Regularity conditions for undiscounted MDPs

When the discount γ is 1, we'll assume:

- There is an **absorbing** state s_0 s.t.
  - s_0 can never be left once entered, and
  - from s_0, no further rewards can be gained.
- The transition probabilities T are **proper**: starting from any state, following any policy will lead to s_0 with probability 1.

Modifying the reward function to provide guidance

To learn a policy for an MDP

$$M = \langle S, A, T, \gamma, R \rangle$$

we could instead run our reinforcement learning algorithm on a transformed MDP

$$M' = \langle S, A, T, \gamma, R' \rangle$$

where

$$R' = R + F$$

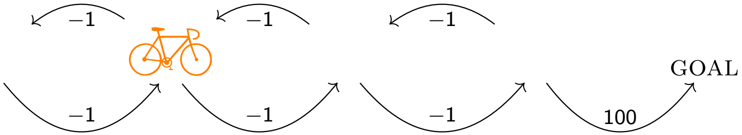
is the transformed reward function, and

$$F : S \times A \times S \to \mathbb{R}$$

is the **shaping reward function**.
When will an optimal (or good) policy for M' also be optimal (or good) for M?

Difficulties in reward shaping

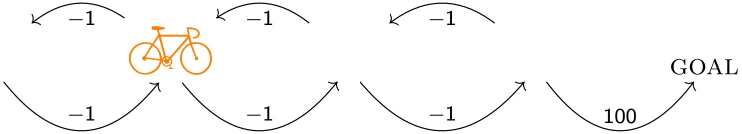
Consider this (undiscounted) problem:



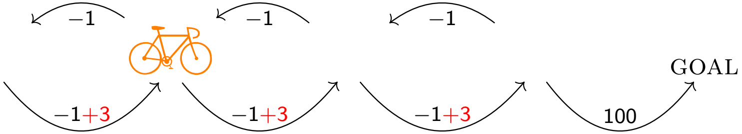
How can we modify the reward function to make the agent more quickly learn to move rightward to the goal?

Difficulties in reward shaping

Consider this (undiscounted) problem:



What if we give extra reward for going in the right direction?



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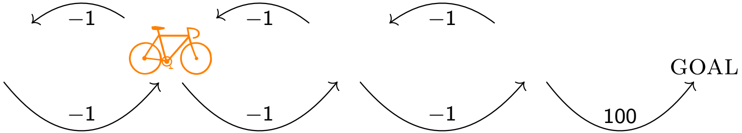

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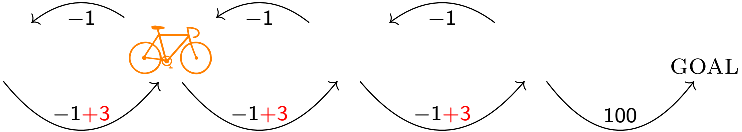


Problem: it's now better for the bicycle to try to go in a circle than to go the goal.

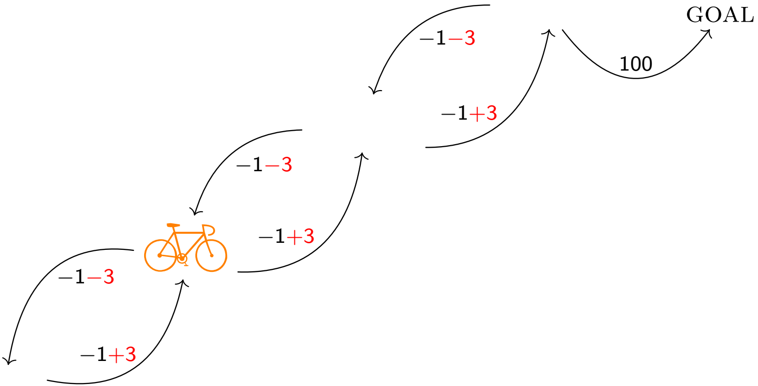
This problem isn't just a contrived artificial example.

Consider this description of work on a (more complicated) bicycle driving domain:

> *In our first experiments we rewarded the agent for driving towards the goal but did not punish it for driving away from it. Consequently the agent drove in circles with a radius of 20–50 meters around the starting point. Such behavior was actually rewarded by the reinforcement function [...]*
>
> — Randløv and Alstrøm (1998)

Idea: use a potential function

Associate a **potential** value $\Phi(s)$ to each state s, and add to the reward of a transition the difference of potentials.



$\Phi(s_1) = 0 \quad \Phi(s_2) = 3 \quad \Phi(s_3) = 6 \quad \Phi(s_4) = 9 \quad \Phi(s_0) = 9$

Definition

A shaping reward function $F : S \times A \times S \to \mathbb{R}$ is **potential-based** if there exists $\Phi : S \to \mathbb{R}$ s.t.

$$F(s, a, s') = \gamma\Phi(s') - \Phi(s)$$

for all $s \neq s_0, a, s'$.

Theorem

If F is a potential-based shaping function, then every optimal policy in $M' = \langle S, A, T, \gamma, R + F\rangle$ will also be an optimal policy in $M = \langle S, A, T, \gamma, R\rangle$ (and vice versa).

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Q^*_M satisfies the Bellman equation:

$$Q^*_M(s, a) = \mathbb{E}_{s' \sim P_{sa}} \left[R(s, a, s') + \gamma \max_{a' \in A} Q^*_M(s', a') \right]$$

Let's subtract $\Phi(s)$ from both sides:

$$\begin{aligned} Q^*_M(s, a) - \Phi(s) &= \mathbb{E}_{s' \sim P_{sa}} \left[R(s, a, s') + \gamma \max_{a' \in A} Q^*_M(s', a') \right] - \Phi(s) \\ &= \mathbb{E}_{s' \sim P_{sa}} \left[R(s, a, s') + \gamma \Phi(s') + \gamma \max_{a' \in A} (Q^*_M(s', a') - \Phi(s')) \right] - \Phi(s) \\ &= \mathbb{E}_{s' \sim P_{sa}} \left[R(s, a, s') + \gamma \Phi(s') - \Phi(s) + \gamma \max_{a' \in A} \left(Q^*_M(s', a') - \Phi(s') \right) \right] \end{aligned}$$

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So $Q^*_M(s, a) - \Phi(s)$ is equal to

$$\mathbb{E}_{s' \sim P_{sa}} \left[R(s, a, s') + \gamma\Phi(s') - \Phi(s) + \gamma \max_{a' \in A} \left(Q^*_M(s', a') - \Phi(s') \right) \right].$$

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Let

$$\hat{Q}_{M'}(s, a) \coloneqq Q^*_M(s, a) - \Phi(s).$$

and recall that

$$F(s, a, s') = \gamma \Phi(s') - \Phi(s).$$

Therefore,

$$\begin{aligned} \hat{Q}_{M'}(s, a) &= \mathbb{E}_{s' \sim P_{sa}} \left[R(s, a, s') + F(s, a, s') + \gamma \max_{a' \in A} \left(\hat{Q}_{M'}(s', a') \right) \right] \\ &= \mathbb{E}_{s' \sim P_{sa}} \left[R'(s, a, s') + \gamma \max_{a' \in A} \left(\hat{Q}_{M'}(s', a') \right) \right] \end{aligned}$$

Theorem

If F is a potential-based shaping function, then every optimal policy in $M' = \langle S, A, T, \gamma, R + F\rangle$ will also be an optimal policy in $M = \langle S, A, T, \gamma, R\rangle$ (and vice versa).

$$\begin{aligned}\hat{Q}_{M'}(s,a) &= \mathbb{E}_{s' \sim P_{sa}}\left[R(s,a,s') + F(s,a,s') + \gamma \max_{a' \in A}\left(\hat{Q}_{M'}(s',a')\right)\right] \\ &= \mathbb{E}_{s' \sim P_{sa}}\left[R'(s,a,s') + \gamma \max_{a' \in A}\left(\hat{Q}_{M'}(s',a')\right)\right]\end{aligned}$$

This is the Bellman equation for M', so

$$\hat{Q}_{M'} = Q^*_{M'}.$$

(In the undiscounted case, $s = s_0$ has to be treated as a special case.)

Corollary

Suppose $F(s, a, s') = \gamma\Phi(s') - \Phi(s)$ (and, if $\gamma = 1$, that $\Phi(s_0) = 0$). Then, for all s, a:

$$Q^*_{M'}(s, a) = Q^*_M(s, a) - \Phi(s) \qquad V^*_{M'} = V^*_M(s) - \Phi(s)$$

Remark

The identities above actually hold for any policy π:

$$Q^\pi_{M'}(s, a) = Q^\pi_M(s, a) - \Phi(s) \qquad V^\pi_{M'} = V^\pi_M(s) - \Phi(s)$$

Therefore, potential-based shaping also preserves near-optimal policies.

- Note that setting $\Phi(s) = V^*_M(s)$ would make $V^*_{M'} \equiv 0$, which would make learning easy.
- This suggests that a way to define a good potential function might be to try to approximate $V^*_M(s)$.

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Potential-based reward shaping
: To ensure that good policies for a modified reward function are also good for the original, it suffices to base the rewards on a **potential function**.

Experiments
: Some potential-based shaping functions are evaluated.

A grid world

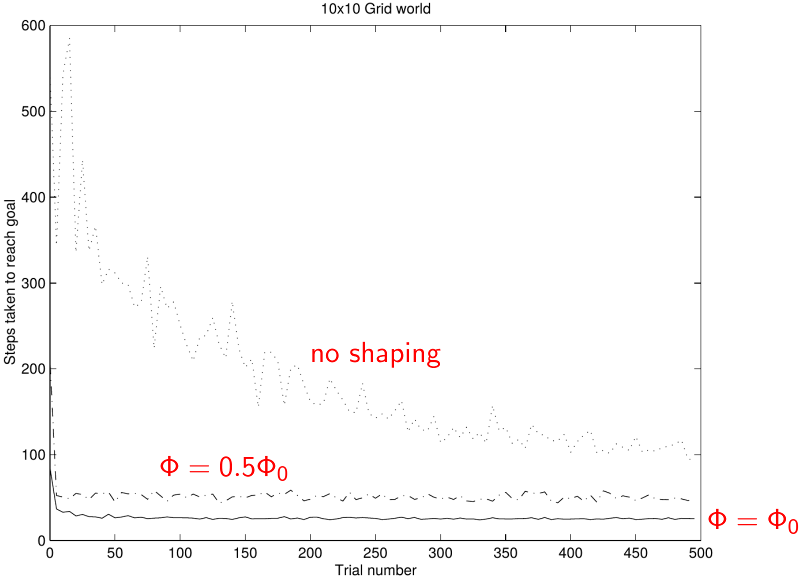
- **States**: an $n \times n$ grid, with start state and (absorbing) goal state in opposite corners.
- **Actions**: can attempt to move in any of the four cardinal directions (N, S, E, W)
- **Transition probabilities**: attempting to move in a direction succeeds with probability 0.8 and goes in a random direction otherwise
- **Discount factor**: $\gamma = 1$ (no discounting)
- **Reward function**: -1 per step

Finding a potential function to approximate V_M^*

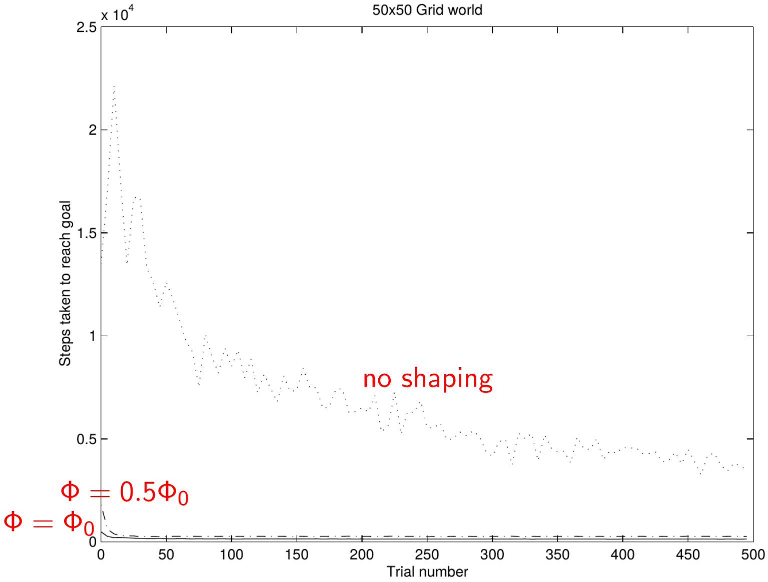
- From most states, trying to move towards the goal could be expected to make roughly 0.8 units of progress.
- Therefore, one estimate of the value function is

$$\Phi_0(s) = -\text{MANHATTAN}(s, \text{GOAL})/0.8$$

- The experiments try using Φ_0 and $0.5\Phi_0$ as potential functions.



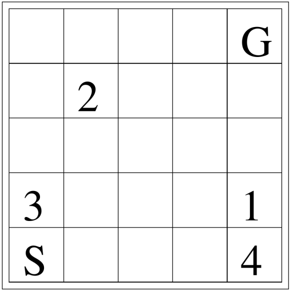
Graph from Figure 1(a) (with red labels added)



Graph from Figure 1(b) (with red labels added)

Grid world with flags

- Extend the grid world so that numbered flags have to be picked up in order.
- The state space is enlarged to keep track of the flags picked up so far.



The agent (S) needs to go to 1, 2, 3, 4, G in order.[1]

[1] Image taken from Figure 2(a)

Grid world with flags

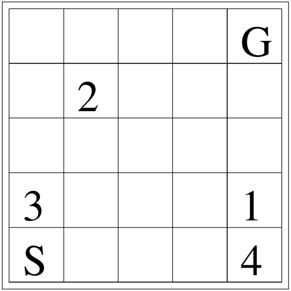
An estimate of the value function is

$$\Phi_0(s) = -\frac{(5 - n - 0.5)}{5} t$$

where

- n is the number of subgoals that have been accomplished in state s, and
- t is an estimate of the number of steps needed to reach G directly.

Experiments were done with Φ_0 and also a function Φ_1 which was a more fine-tuned estimate.



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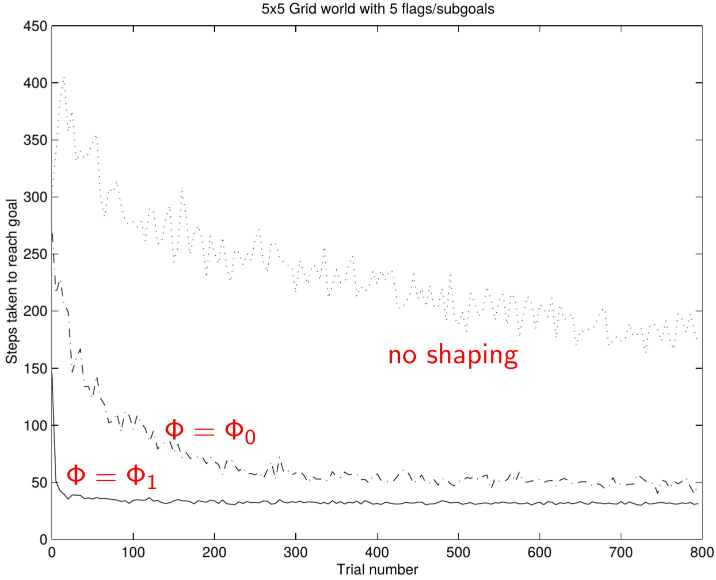

Graph from Figure 2(b) (with red labels added)

Conclusion

We've seen that

- Reward shaping can change what the optimal policy is.
- But, using potential-based shaping functions guarantees that the optimal policy will not be changed.
- The idea of potential functions can help us find useful shaping functions in practice.

References

Jette Randløv and Preben Alstrøm. Learning to drive a bicycle using reinforcement learning and shaping. In *Proceedings of the Fifteenth International Conference on Machine Learning*, ICML '98, pages 463–471, San Francisco, CA, USA, 1998. Morgan Kaufmann Publishers Inc.