Hash-Based Indexes

Chapter 11

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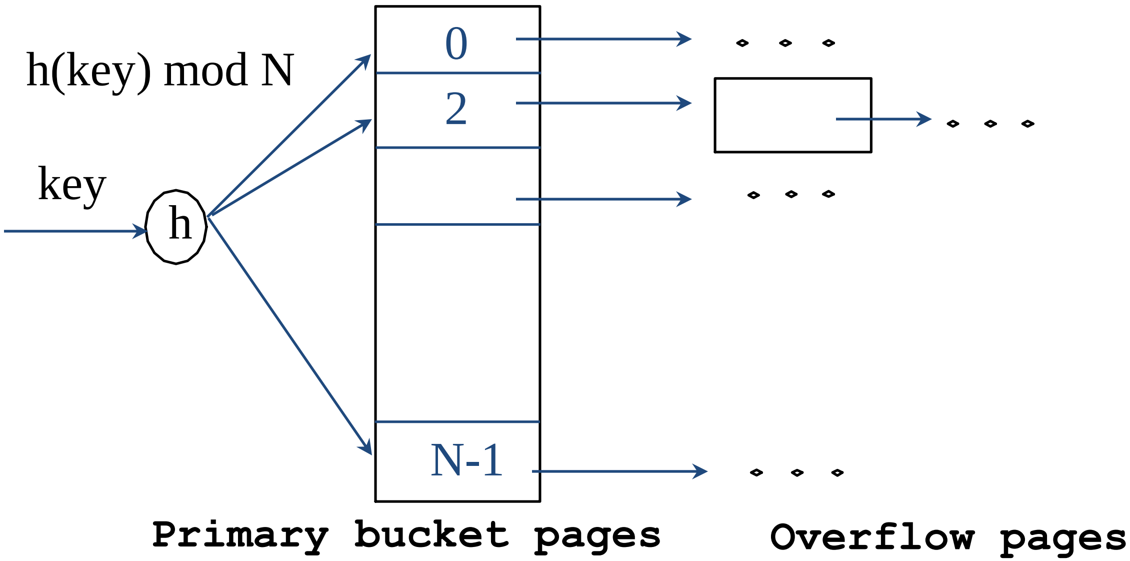
Introduction

- *As for any index, 3 alternatives for data entries* **k***:
  - Data record with key value **k**
  - <**k**, rid of data record with search key value **k**>
  - <**k**, list of rids of data records with search key **k**>
  - Choice orthogonal to the *indexing technique*
- *Hash-based* indexes are best for *equality selections*. ***Cannot*** support range searches.
- Static and dynamic hashing techniques exist;

Static Hashing

- # primary pages fixed, allocated sequentially, never de-allocated; overflow pages if needed.
- **h**(*k*) mod N = bucket to which data entry with key *k* belongs. (N = # of buckets)



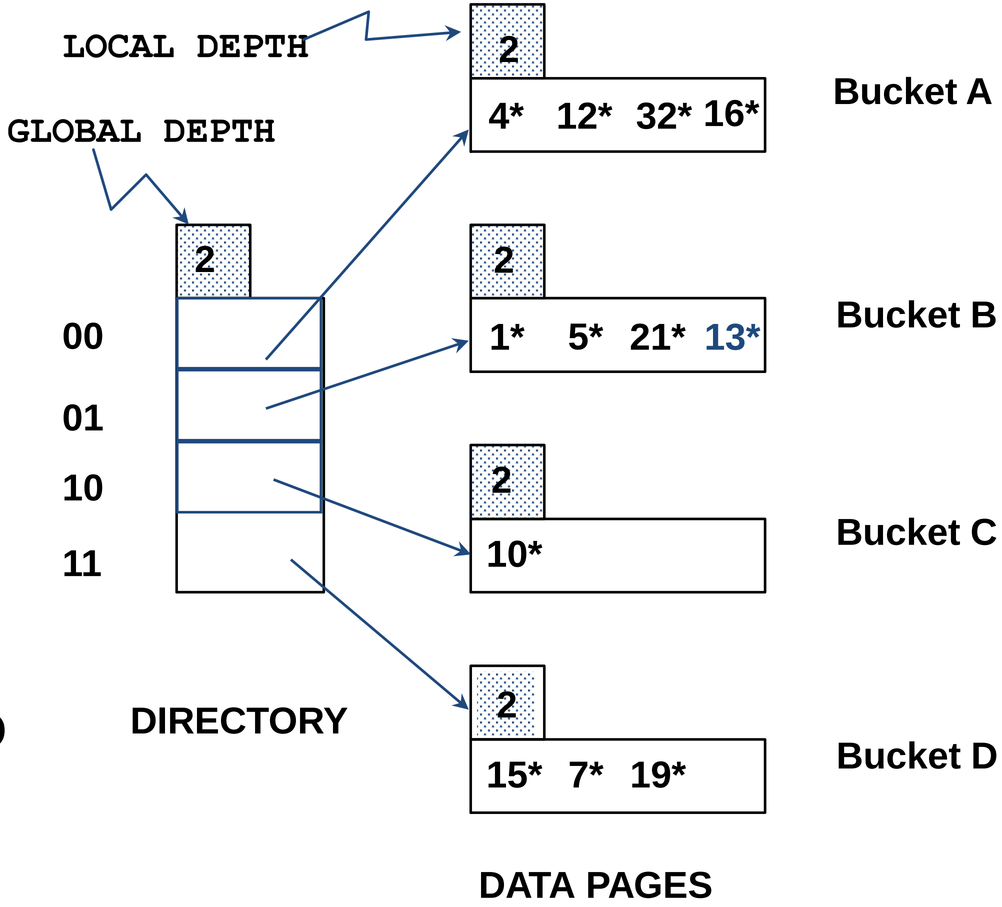
Static Hashing (Contd.)

- Buckets contain *data entries*.
- Hash fn works on *search key* field of record *r.* Must distribute values over range 0 ... M-1.
  - **h**(*key*) = (a * *key* + b) usually works well.
  - a and b are constants; lots known about how to tune **h**.
- Long overflow chains can develop and degrade performance.
  - *Extendible* and *Linear Hashing*: Dynamic techniques to fix this problem.

Extendible Hashing

- Situation: Bucket (primary page) becomes full. Why not re-organize file by *doubling* # of buckets?
  - Reading and writing all pages is expensive!
  - *Idea*: Use *directory of pointers to buckets*, double # of buckets by *doubling the directory,* splitting just the bucket that overflowed!
  - Directory much smaller than file, so doubling it is much cheaper. Only one page of data entries is split. *No overflow page*!
  - Trick lies in how hash function is adjusted!

Example

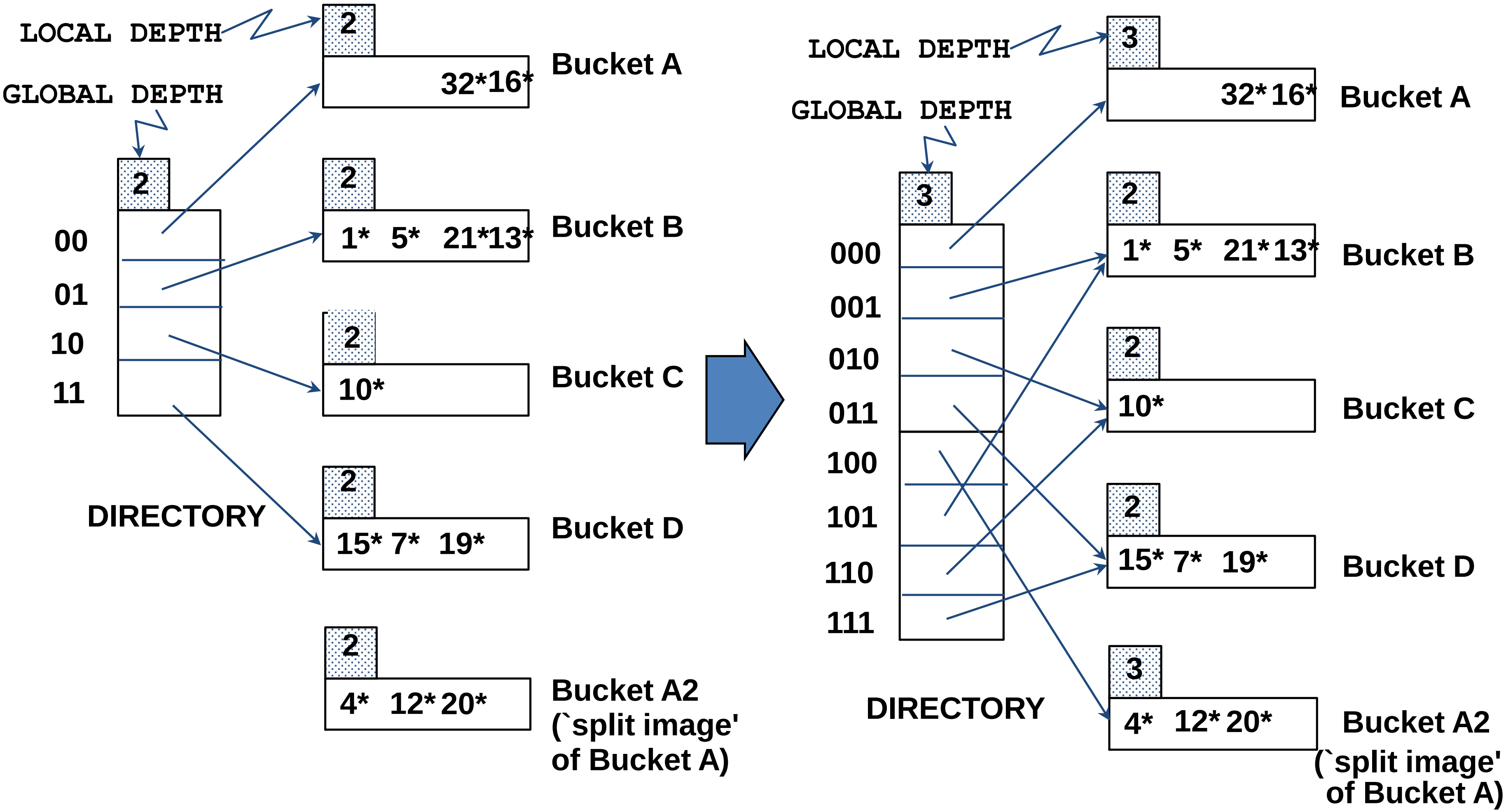


- Directory is array of size 4.
- To find bucket for *r*, take last \`*global depth*' # bits of **h**(*r*); we denote *r* by **h**(*r*).
  - If **h**(*r*) = 5 = binary 101, it is in bucket pointed to by 01.

❖ **Insert**: If bucket is full, *split* it (*allocate new page, re-distribute*).

❖ *If necessary*, double the directory. (As we will see, splitting a bucket does not always require doubling; we can tell by comparing *global depth* with *local depth* for the split bucket.)

Insert **h**(r)=20 (Causes Doubling)



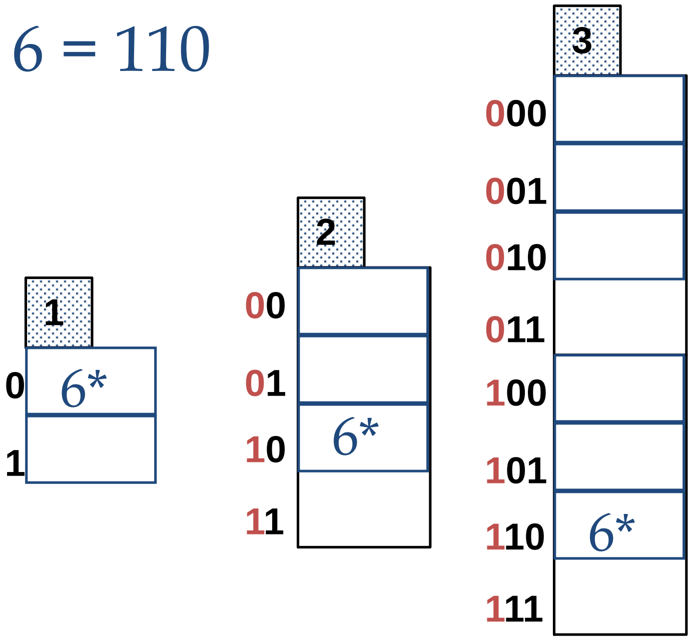
Points to Note

- 20 = binary 10100. Last **2** bits (00) tell us *r* belongs in A or A2. Last **3** bits needed to tell which.
  - *Global depth of directory*: Max # of bits needed to tell which bucket an entry belongs to.
  - *Local depth of a bucket*: # of bits used to determine if an entry belongs to this bucket.
- When does bucket split cause directory doubling?
  - Before insert, *local depth* of bucket = *global depth*. Insert causes *local depth* to become > *global depth*; directory is doubled by *copying it over* and `fixing’ pointer to split image page. (Use of least significant bits enables efficient doubling via copying of directory!)

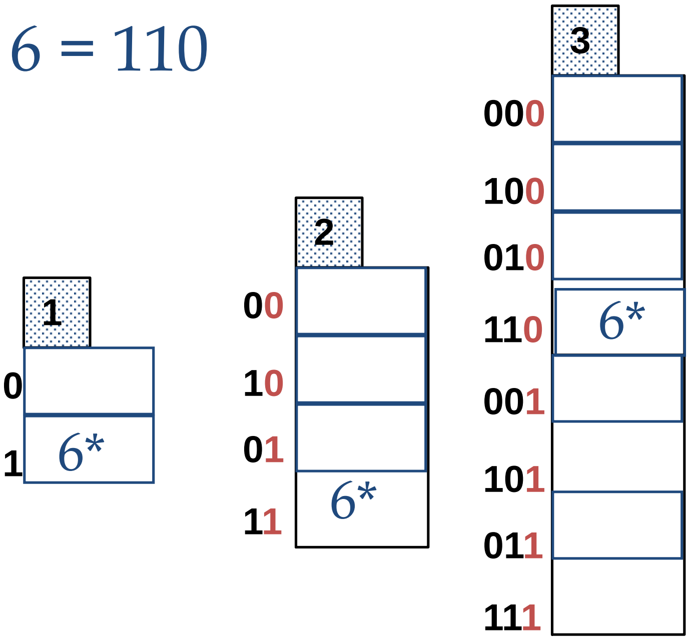
Directory Doubling

Why use least significant bits in directory?
⇔ Allows for doubling via copying!

6 = 110



6 = 110



Least Significant vs. Most Significant

Comments on Extendible Hashing

- If directory fits in memory, equality search answered with one disk access; else two.
  - 100MB file, 100 bytes/rec, 4K pages contains 1,000,000 records (as data entries) and 25,000 directory elements; chances are high that directory will fit in memory.
  - Directory grows in spurts, and, if the distribution *of hash values* is skewed, directory can grow large.
  - Multiple entries with same hash value cause problems!
- **Delete**: If removal of data entry makes bucket empty, can be merged with `split image'. If each directory element points to same bucket as its split image, can halve directory.

Linear Hashing

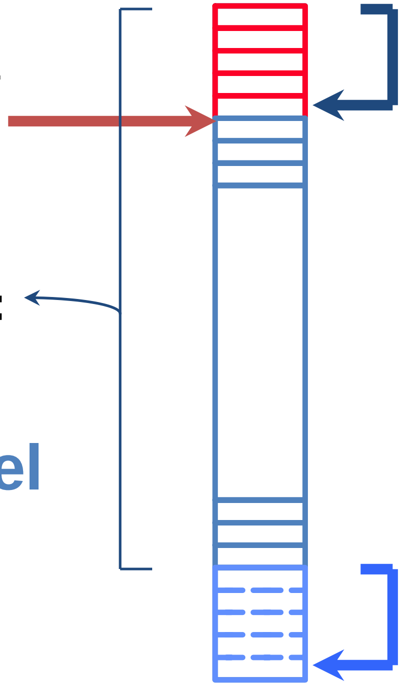
- This is another dynamic hashing scheme, an alternative to Extendible Hashing.
- LH handles the problem of long overflow chains without using a directory, and handles duplicates.
- *Idea*: Use a family of hash functions $\mathbf{h}_0$, $\mathbf{h}_1$, $\mathbf{h}_2$, ...
  - $\mathbf{h}_i(key) = \mathbf{h}(key) \bmod (2^i N)$; N = initial # buckets
  - **h** is some hash function (range is *not* 0 to N-1)
  - If $N = 2^{d0}$, for some *d0*, $\mathbf{h}_i$ consists of applying **h** and looking at the last *di* bits, where $di = d0 + i$.
  - $\mathbf{h}_{i+1}$ doubles the range of $\mathbf{h}_i$ (similar to directory doubling)

Linear Hashing (Contd.)

- Directory avoided in LH by using overflow pages, and choosing bucket to split round-robin.
  - Splitting proceeds in `rounds'. Round ends when all N_R initial (for round *R*) buckets are split. Buckets 0 to *Next-1* have been split; *Next* to N_R yet to be split.
  - Current round number is *Level*.
  - **Search:** To find bucket for data entry *r,* find $\mathbf{h}_{Level}(r)$*:*
    - If $\mathbf{h}_{Level}(r)$ in range `*Next* to N_R' , *r* belongs here.
    - Else, r could belong to bucket $\mathbf{h}_{Level}(r)$ or bucket $\mathbf{h}_{Level}(r) + N_R$; must apply $\mathbf{h}_{Level+1}(r)$ to find out.

Overview of LH File

- In the middle of a round.



Bucket to be split

Next

Buckets that existed at the beginning of this round: this is the range of

h_{Level}

Buckets split in this round: If h_{Level} (search key value) is in this range, must use $h_{Level+1}$ (search key value) to decide if entry is in `split image' bucket.

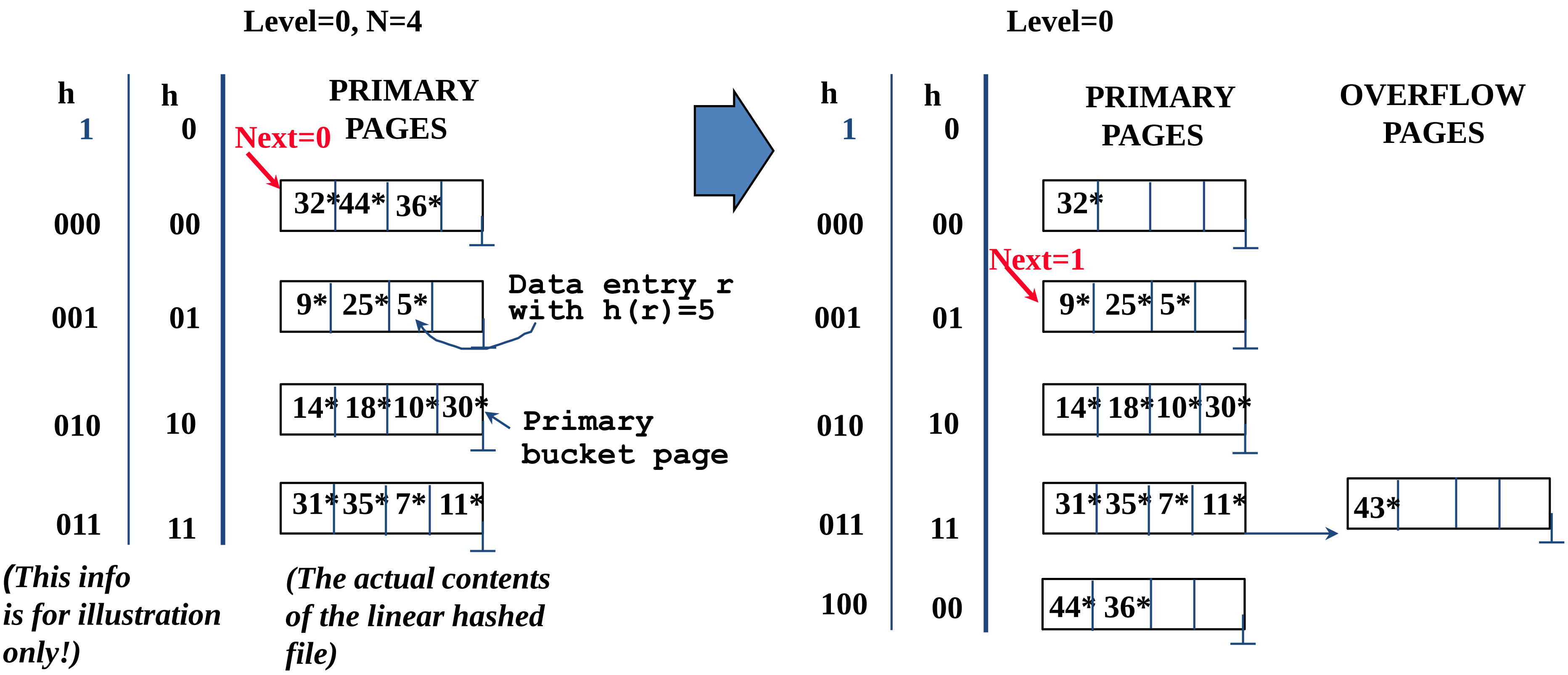
`split image' buckets: created (through splitting of other buckets) in this round

Linear Hashing (Contd.)

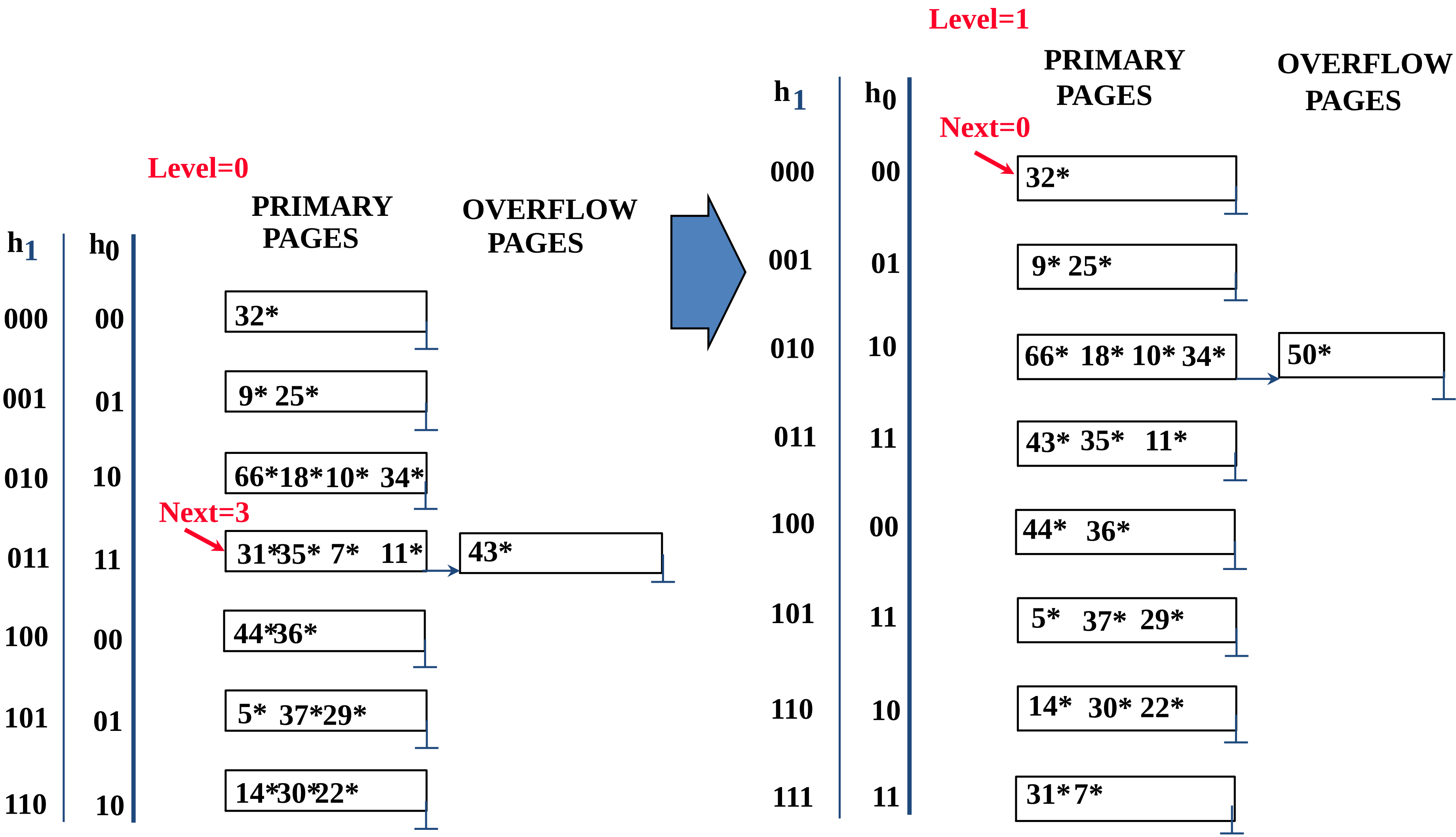
- **Insert**: Find bucket by applying $\mathbf{h}_{Level}$ / $\mathbf{h}_{Level+1}$:
  - If bucket to insert into is full:
    - Add overflow page and insert data entry.
    - (*Maybe*) Split *Next* bucket and increment *Next*.
- Can choose any criterion to `trigger' split.
- Since buckets are split round-robin, long overflow chains don't develop!
- Doubling of directory in Extendible Hashing is similar; switching of hash functions is *implicit* in how the # of bits examined is increased.

Example of Linear Hashing

- On split, **h**$_{\text{Level+1}}$ is used to re-distribute entries.



Example: End of a Round



Level=0

h1	h0	PRIMARY PAGES	OVERFLOW PAGES
000	00	32*	
001	01	9* 25*	
010	10	66*18*10* 34*	
011	11	Next=3 31*35* 7* 11*	43*
100	00	44*36*	
101	01	5* 37*29*	
110	10	14*30*22*	

Level=1

h1	h0	PRIMARY PAGES	OVERFLOW PAGES
000	00	Next=0 32*	
001	01	9* 25*	
010	10	66* 18* 10* 34*	50*
011	11	43* 35* 11*	
100	00	44* 36*	
101	11	5* 37* 29*	
110	10	14* 30* 22*	
111	11	31* 7*	

Summary

- Hash-based indexes: best for equality searches, cannot support range searches.
- Static Hashing can lead to long overflow chains.
- Extendible Hashing avoids overflow pages by splitting a full bucket when a new data entry is to be added to it. (*Duplicates may require overflow pages.*)
  - Directory to keep track of buckets, doubles periodically.
  - Can get large with skewed data; additional I/O if this does not fit in main memory.

Summary (Contd.)

- Linear Hashing avoids directory by splitting buckets round-robin, and using overflow pages.
  - Overflow pages not likely to be long.
  - Duplicates handled easily.
  - Space utilization could be lower than Extendible Hashing, since splits not concentrated on `dense’ data areas.
    - Can tune criterion for triggering splits to trade-off slightly longer chains for better space utilization.
- For hash-based indexes, a *skewed* data distribution is one in which the *hash values* of data entries are not uniformly distributed!

External Sorting

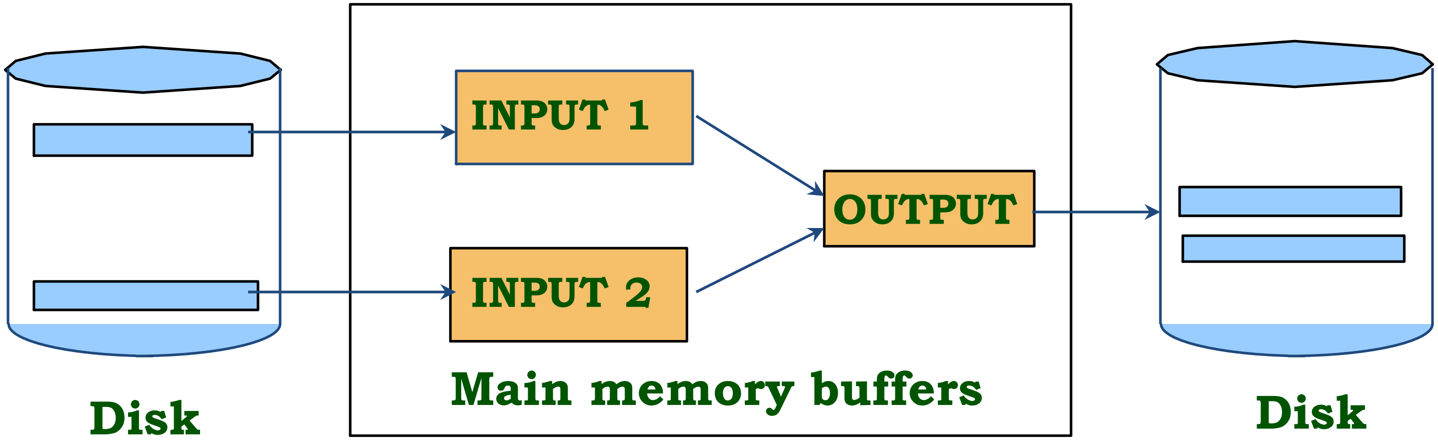
Chapter 13

Why Sort?

- A classic problem in computer science!
- Data requested in sorted order
  - e.g., find students in increasing *gpa* order
- Sorting is first step in *bulk loading* B+ tree index.
- Sorting useful for eliminating *duplicate copies* in a collection of records (Why?)
- *Sort-merge* join algorithm involves sorting.
- Problem: sort 1Gb of data with 1Mb of RAM.
  - why not virtual memory?

2-Way Sort: Requires 3 Buffers

- Pass 1: Read a page, sort it, write it.
  - only one buffer page is used
- Pass 2, 3, …, etc.:
  - three buffer pages used.



Two-Way External Merge Sort

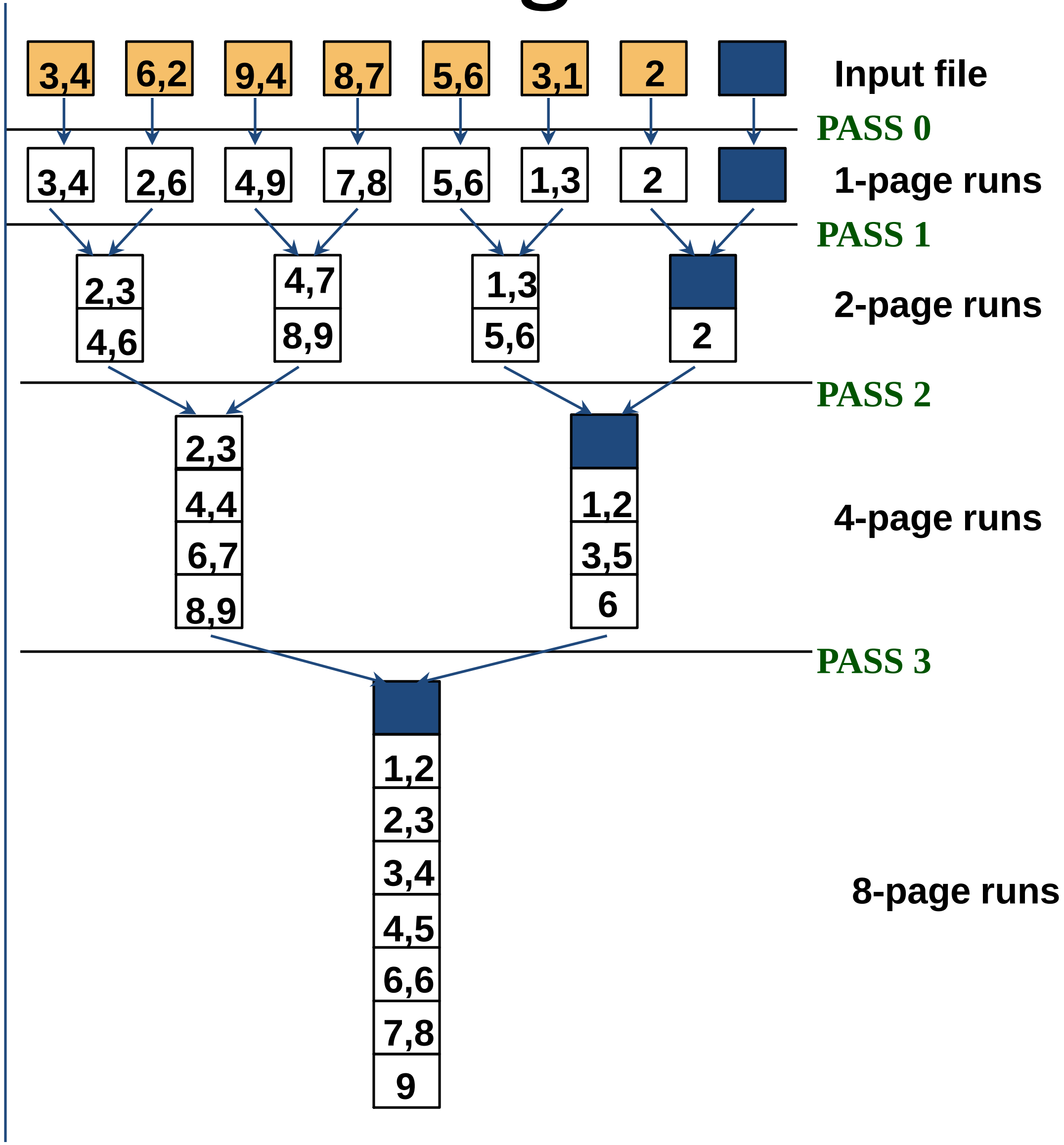
- Each pass we read + write each page in file.
- N pages in the file => the number of passes

$$= \lceil \log_2 N \rceil + 1$$

- So toal cost is:

$$2N\left(\lceil \log_2 N \rceil + 1\right)$$

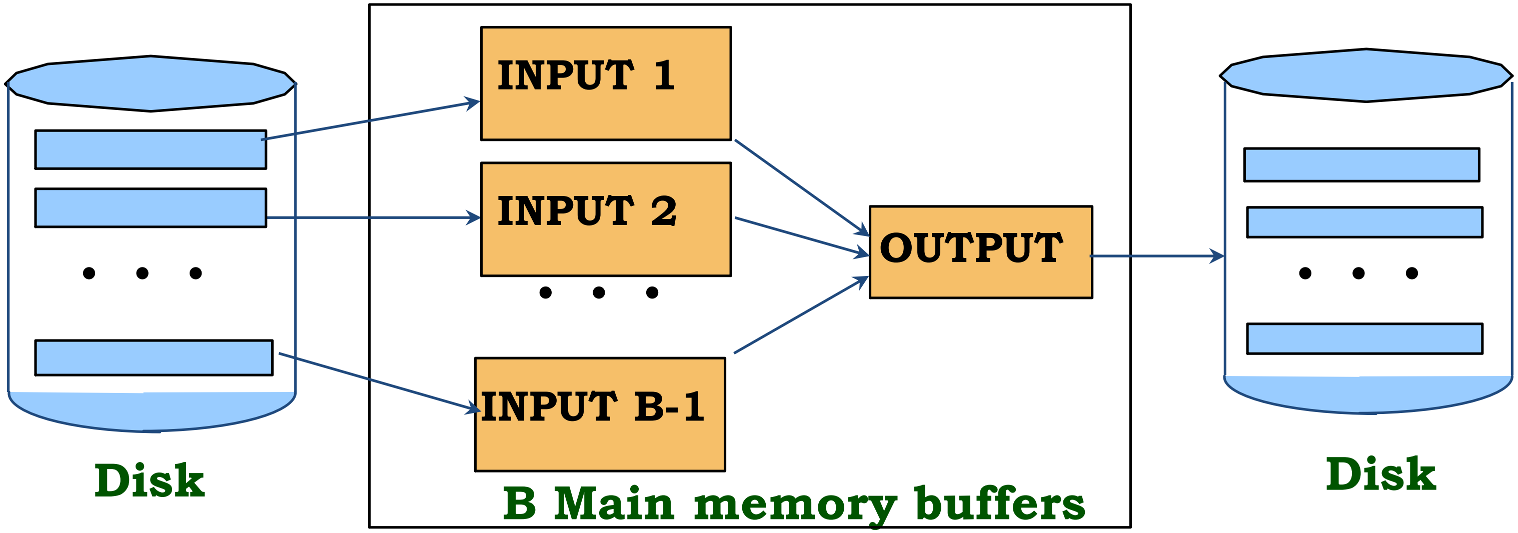
- _Idea:_ ***Divide and conquer:*** sort subfiles and merge



General External Merge Sort

□ ***More than 3 buffer pages. How can we utilize them?***

- To sort a file with *N* pages using *B* buffer pages:
  - Pass 0: use *B* buffer pages. Produce $\lceil N / B \rceil$ sorted runs of *B* pages each.
  - Pass 2, …, etc.: merge *B-1* runs.



Cost of External Merge Sort

- Number of passes:
- Cost = 2N * (# of passes) $1+\lceil \log_{B-1}\lceil N / B \rceil \rceil$
- E.g., with 5 buffer pages, to sort 108 page file:
  - Pass 0: $\lceil 108 / 5 \rceil$ = 22 sorted runs of 5 pages each (last run is only 3 pages)
  - Pass 1: $\lceil 22 / 4 \rceil$ = 6 sorted runs of 20 pages each (last run is only 8 pages)
  - Pass 2: 2 sorted runs, 80 pages and 28 pages
  - Pass 3: Sorted file of 108 pages

Number of Passes of External Sort

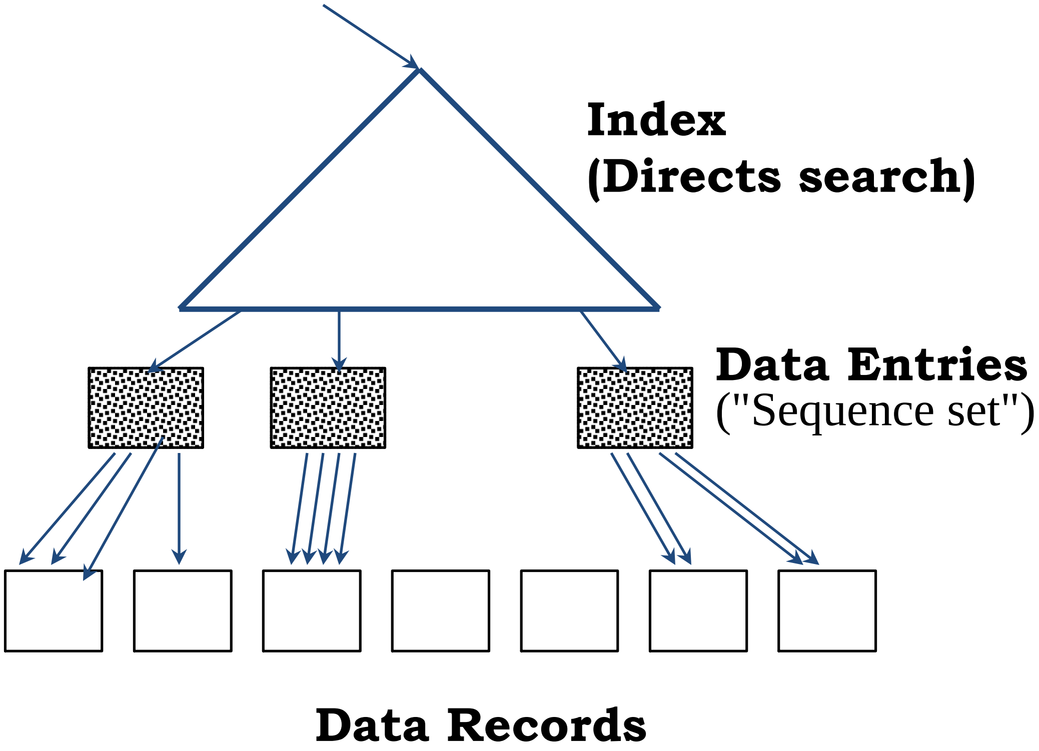
N	B=3	B=5	B=9	B=17	B=129	B=257
100	7	4	3	2	1	1
1,000	10	5	4	3	2	2
10,000	13	7	5	4	2	2
100,000	17	9	6	5	3	3
1,000,000	20	10	7	5	3	3
10,000,000	23	12	8	6	4	3
100,000,000	26	14	9	7	4	4
1,000,000,000	30	15	10	8	5	4

Using B+ Trees for Sorting

- Scenario: Table to be sorted has B+ tree index on sorting column(s).
- Idea: Can retrieve records in order by traversing leaf pages.
- ***Is this a good idea?***
- Cases to consider:
  - B+ tree is clustered ***Good idea!***
  - B+ tree is not clustered ***Could be a very bad idea!***

Clustered B+ Tree Used for Sorting

- Cost: root to the left-most leaf, then retrieve all leaf pages



Index
(Directs search)

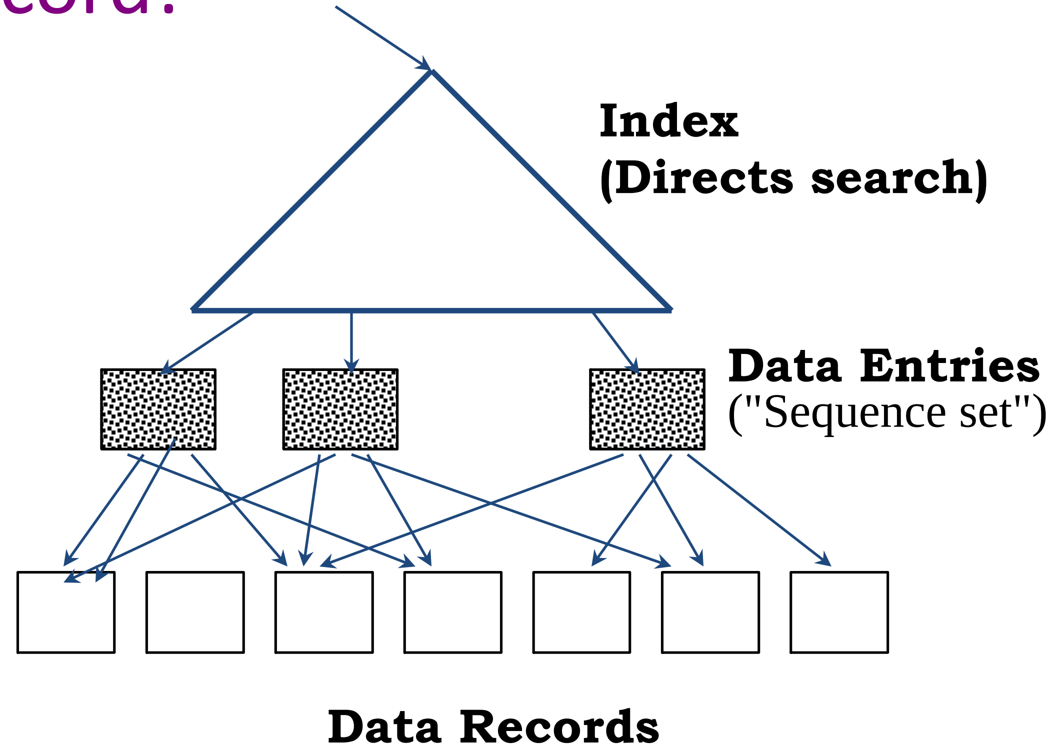
Data Entries
("Sequence set")

Data Records

□ *Always better than external sorting!*

Unclustered B+ Tree Used for Sorting

- For data entries; each data entry contains *rid* of a data record. In general, one I/O per data record!



External Sorting vs. Unclustered Index

N	Sorting	p=1	p=10	p=100
100	200	100	1,000	10,000
1,000	2,000	1,000	10,000	100,000
10,000	40,000	10,000	100,000	1,000,000
100,000	600,000	100,000	1,000,000	10,000,000
1,000,000	8,000,000	1,000,000	10,000,000	100,000,000
10,000,000	80,000,000	10,000,000	100,000,000	1,000,000,000

- ***p*: # of records per page**
- ***B=1,000 and block size=32 for sorting***
- ***p=100 is the more realistic value.***

Summary

- External sorting is important; DBMS may dedicate part of buffer pool for sorting!
- External merge sort minimizes disk I/O cost:
  - Pass 0: Produces sorted ***runs*** of size ***B*** (# buffer pages). Later passes: ***merge*** runs.
  - # of runs merged at a time depends on ***B,*** and ***block size***.
  - Larger block size means less I/O cost per page.
  - Larger block size means smaller # runs merged.
  - In practice, # of runs rarely more than 2 or 3.

Summary, cont.

- Choice of internal sort algorithm may matter:
  - Quicksort: Quick!
  - Heap/tournament sort: slower (2x), longer runs
- The best sorts are wildly fast:
  - Despite 40+ years of research, we’re still improving!
- Clustered B+ tree is good for sorting; unclustered tree is usually very bad.