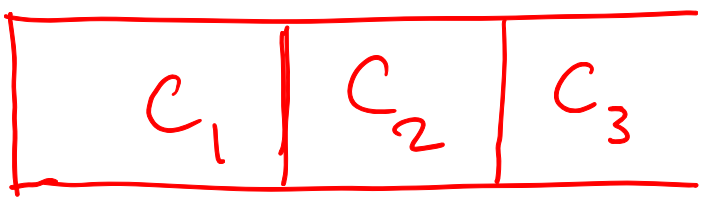
GAC Example

Consider the following CSP:

$$Dom[X] = \{1, 2, 3, 4\} \qquad Dom[Y] = \{1, 2, 3, 4\}$$
$$Dom[Z] = \{1, 2, 3, 4\} \qquad Dom[W] = \{1, 2, 3, 4, 5\}$$
$$C_1(X, Y, Z) : X = Y + Z$$
$$C_2(X, W) : W > X$$
$$C_3(X, Y, Z, W) : W = X + Z + Y$$

Enforce GAC on these constraints, and give the resultant GAC consistent variable domains.

GAC queue



$CurDom[X] = \{1, 2, 3, 4\}$

$CurDom[Y] = \{1, 2, 3, 4\}$

$CurDom[Z] = \{1, 2, 3, 4\}$

$CurDom[W] = \{1, 2, 3, 4, 5\}$

$C_1(X, Y, Z) : X = Y + Z$

$C_2(X, W) : W > X$

$C_3(X, Y, Z, W) : W = X + Z + Y$

C_1	C_2	C_3

- Processing C_3:

$X = 1 - (Y = 1, Z = 1, W = 3)$

$X = 2 - (Y = 1, Z = 1, W = 4)$

$X = 3 - (Y = 1, Z = 1, W = 5)$

$X = 4$ - No support

Similarly, $Y = 4$ and $Z = 4$ have no support

W=1 - No support

W=2 - No support

W=3 - Same support as X=1

W=4 - " " " X=2

W=5 - " " " X=3

$CurDom[X] = \{1, 2, 3\}$

$CurDom[Y] = \{1, 2, 3\}$

$CurDom[Z] = \{1, 2, 3\}$

$CurDom[W] = \{3, 4, 5\}$

$C_1(X, Y, Z) : X = Y + Z$

$C_2(X, W) : W > X$

$C_3(X, Y, Z, W) : W = X + Z + Y$

- Processing C_2:

X=1 - (W=3)

X=2 - (W=3)

X=3 - (W=4)

W=3 - same support as X=1

W=4 - " " " X=3

W=5 - (X=1)

.

$CurDom[X] = \{1, 2, 3\}$ → C_3 and C_2 on queue

$CurDom[Y] = \{1, 2, 3\}$

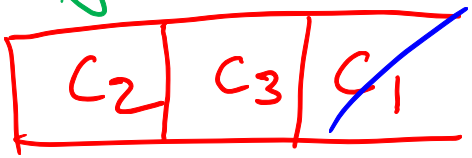
$CurDom[Z] = \{1, 2, 3\}$

$CurDom[W] = \{3, 4, 5\}$

$C_1(X, Y, Z) : X = Y + Z$

$C_2(X, W) : W > X$

$C_3(X, Y, Z, W) : W = X + Z + Y$



- Processing C_1:

X=1 - No support

X=2 - (Y=1, Z=1)

X=3 - (Y=1, Z=2)

Y=1 - Same support as X=2

Y=2 - (X=3, Z=1)

Y=3 - No support

$Z=1$ - Same support as $X=2$

$Z=2$ - " " " $X=3$

$Z=3$ - No support

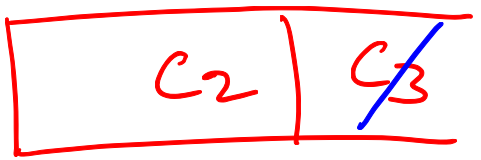
$CurDom[X] = \{2, 3\}$ $CurDom[Y] = \{1, 2\}$

$CurDom[Z] = \{1, 2\}$ $CurDom[W] = \{3, 4, 5\}$

$C_1(X, Y, Z) : X = Y + Z$

$C_2(X, W) : W > X$

$C_3(X, Y, Z, W) : W = X + Z + Y$



- Processing C_3:

$X = 2$ – $(Y = 1, Z = 1, W = 4)$

$X = 3$ – $(Y = 1, Z = 1, W = 5)$

$Y = 1$ – Same support as $X = 2$

$Y = 2$ – $(X = 2, Z = 1, W = 5)$

$Z=1$ - Same support as $X=2$

$Z=2$ - $(X=2, Y=1, W=5)$

.

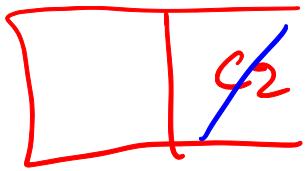
$W=3$ - No support

$W=4$ - Same support as $X=2$

$W=5$ - " " " $Z=2$

$$CurDom[X] = \{2, 3\} \qquad CurDom[Y] = \{1, 2\}$$
$$CurDom[Z] = \{1, 2\} \qquad CurDom[W] = \{4, 5\}$$

$$C_1(X, Y, Z) : X = Y + Z$$
$$C_2(X, W) : W > X$$
$$C_3(X, Y, Z, W) : W = X + Z + Y$$



- Processing C_2:

$X = 2 - (W = 4)$ $\qquad$ $W = 4 - (X = 2)$

$X = 3 - (W = 4)$ $\qquad$ $W = 5 - (X = 2)$

.

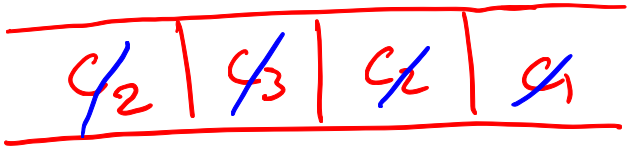
$CurDom[X] = \{2, 3\}$ $CurDom[Y] = \{1, 2\}$

$CurDom[Z] = \{1, 2\}$ $CurDom[W] = \{4, 5\}$

$C_1(X, Y, Z) : X = Y + Z$

$C_2(X, W) : W > X$

$C_3(X, Y, Z, W) : W = X + Z + Y$



- Branch on X:
  $X = 2$

GAC on C_1 => $Dom[X] = \{1\}$, $Dom[Z] = \{1\}$

GAC on C_2 => No changes

GAC on C_3 => $Dom[W] = \{4\}$ → must put C_2 on queue

GAC on C_2 => No changes.

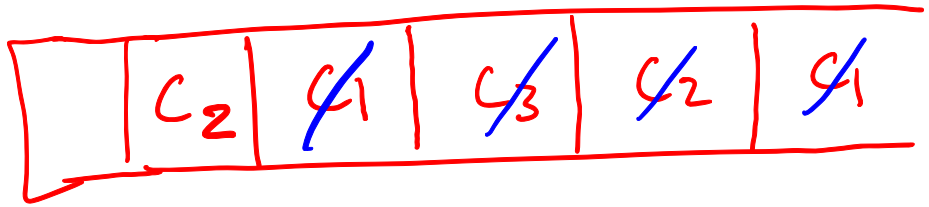
$CurDom[X] = \{2, 3\}$ $CurDom[Y] = \{1, 2\}$

$CurDom[Z] = \{1, 2\}$ $CurDom[W] = \{4, 5\}$

$C_1(X, Y, Z) : X = Y + Z$

$C_2(X, W) : W > X$

$C_3(X, Y, Z, W) : W = X + Z + Y$



- Branch on X:
  $X = 3$

GAC on C_1 => No changes

GAC on C_2 => " "

GAC on C_3 => Dom[Y] = {1} Dom[Z] = {1}

C_1 on queue

Dom[W] = {5} => C_2 on queue

GAC on C_1 => Dom[Y] = {} DWO!

Note: No solution for X=3, but GAC enforce didn't prune it.

GAC Example

C1(V1,V2,V3)

V1	V2	V3
A	B	C
B	A	C
A	A	B

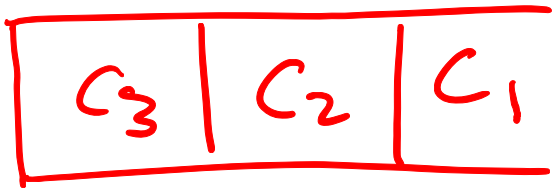
C2(V1,V3,V4,V5)

V1	V3	V4	V5
A	A	A	A
A	B	C	B
B	C	B	B
C	A	B	C
C	B	A	B

C3(V2,V3,V5)

V2	V3	V5
A	A	A
A	B	C
B	C	B
C	A	B
C	B	A

$$CurDom[V_1] = CurDom[V2] = ... = CurDom[V_5] = \{A, B, C\}$$



C1(V1,V2,V3)

V1	V2	V3
A	B	C
B	A	C
A	A	B

C2(V1,V3,V4,V5)

V1	V3	V4	V5
~~A~~	~~A~~	~~A~~	~~A~~
A	B	C	B
B	C	B	B
~~C~~	~~A~~	~~B~~	~~C~~
~~C~~	~~B~~	~~A~~	~~B~~

C3(V2,V3,V5)

V2	V3	V5
~~A~~	~~A~~	~~A~~
A	B	C
B	C	B
~~C~~	~~A~~	~~B~~
C	B	A

$CurDom[V_1] = \{A, B, \cancel{C}\}$ $CurDom[V_2] = \{A, B, \cancel{C}\}$

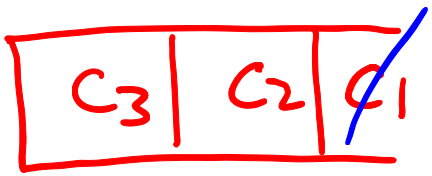
$CurDom[V_3] = \{\cancel{A}, B, C\}$ $CurDom[V_4] = \{A, B, C\}$ $CurDom[V_5] = \{A, B, C\}$

Processing C_1:

$V_1 = C$ - No supports

$V_2 = C$ - " "

$V_3 = A$ - " "



C1(V1,V2,V3)

V1	V2	V3
A	B	C
B	A	C
A	A	B

C2(V1,V3,V4,V5)

V1	V3	V4	V5
A	A	A	A
A	B	C	B
B	C	B	B
C	A	B	C
C	B	A	B

C3(V2,V3,V5)

V2	V3	V5
~~A~~	~~A~~	~~A~~
~~A~~	~~B~~	~~C~~
B	C	B
~~C~~	~~A~~	~~B~~
~~C~~	~~B~~	~~A~~

$CurDom[V_1] = \{A, B\}$ $CurDom[V_2] = \{A, B\}$

$CurDom[V_3] = \{B, C\}$ $CurDom[V_4] = \{\cancel{A}, B, C\}$ $CurDom[V_5] = \{\cancel{A}, B, \cancel{C}\}$

Processing C_2:

$V_4 = A$ - No support

$V_5 = A$ - " "

$V_5 = C$ - " "

C_3	~~C_2~~

.

C1(V1,V2,V3)

V1	V2	V3
A	B	C
~~B~~	~~A~~	~~C~~
~~A~~	~~A~~	~~B~~

C2(V1,V3,V4,V5)

V1	V3	V4	V5
A	A	A	A
A	B	C	B
B	C	B	B
C	A	B	C
C	B	A	B

C3(V2,V3,V5)

V2	V3	V5
A	A	A
A	B	C
B	C	B
C	A	B
C	B	A

$CurDom[V_1] = \{A, B\}$ $CurDom[V_2] = \{A, B\}$

$CurDom[V_3] = \{B, C\}$ $CurDom[V_4] = \{B, C\}$ $CurDom[V_5] = \{B\}$

Processing C_3:

$V_2 = A$ - No support

$V_3 = B$ - " "

	C_1	C_3

.

C1(V1,V2,V3)

V1	V2	V3
A	B	C
B	A	C
A	A	B

C2(V1,V3,V4,V5)

V1	V3	V4	V5
A	A	A	A
A	B	C	B
~~B~~	~~C~~	~~B~~	~~B~~
C	A	B	C
C	B	A	B

C3(V2,V3,V5)

V2	V3	V5
A	A	A
A	B	C
B	C	B
C	A	B
C	B	A

$CurDom[V_1] = \{A, B\}$ $CurDom[V_2] = \{B\}$

$CurDom[V_3] = \{C\}$ $CurDom[V_4] = \{B, C\}$ $CurDom[V_5] = \{B\}$

Processing C_1:

$V_1 = B$ - No support

C_2	~~C_1~~

.

- C1(V1,V2,V3)

V1	V2	V3
A	B	C
B	A	C
A	A	B

- C2(V1,V3,V4,V5)

V1	V3	V4	V5
A	A	A	A
A	B	C	B
B	C	B	B
C	A	B	C
C	B	A	B

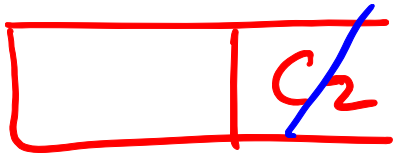
- C2(V2,V3,V5)

V2	V3	V5
A	A	A
A	B	C
B	C	B
C	A	B
C	B	A

$CurDom[V_1] = \{A\}$ $CurDom[V_2] = \{B\}$

$CurDom[V_3] = \{C\}$ $CurDom[V_4] = \{B, C\}$ $CurDom[V_5] = \{B\}$

Processing C_2:



$V_4 = B$ - No support

$V_3 = C$ - " " => $CurDom[V_3] = \{\}$ DWO!

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