The Relational Model

csc343, Introduction to Databases
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Recap

- The relational model is based on the concept of a relation or table.
- Two example relations:

<table>
<thead>
<tr>
<th>Teams</th>
<th>Name</th>
<th>Home Field</th>
<th>Coach</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Rangers</td>
<td>Runnymede CI</td>
<td>Tarvo Sinervo</td>
</tr>
<tr>
<td></td>
<td>Ducks</td>
<td>Humber Public</td>
<td>Maeve Mahar</td>
</tr>
<tr>
<td></td>
<td>Choppers</td>
<td>High Park</td>
<td>Tom Cole</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Games</th>
<th>Home team</th>
<th>Away team</th>
<th>Home goals</th>
<th>Away goals</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Rangers</td>
<td>Ducks</td>
<td>3</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>Ducks</td>
<td>Choppers</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>Rangers</td>
<td>Choppers</td>
<td>4</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>Choppers</td>
<td>Ducks</td>
<td>0</td>
<td>5</td>
</tr>
</tbody>
</table>
Relations in Math

- A domain is a set of values.
- Suppose $D_1, D_2, \ldots, D_n$ are domains.
  - The **Cartesian product** $D_1 \times D_2 \times \ldots \times D_n$ is the set of all tuples $<d_1, d_2, \ldots, d_n>$ such that $d_1 \in D_1, \ d_2 \in D_2, \ \ldots, \ d_n \in D_n$.
  - I.e., every combination of a value from $D_1$, a value from $D_2$ etc.

- A **(mathematical) relation** on $D_1, D_2, \ldots, D_n$ is a subset of the Cartesian product.
Example

- Example of a mathematical relation:
  - Let $A = \{p, q, r, s\}$, $B = \{1, 2, 3\}$ and $C = \{100, 200\}$.
  - $R = \{<q, 2, 100>, <s, 3, 200>, <p, 1, 200>\}$ is a relation on $A$, $B$, $C$.

- Our database tables are relations too.

- Example:

  $\{<\text{Rangers}, \text{Ducks}, 3, 0>, <\text{Ducks}, \text{Choppers}, 1, 1>,$
  $<\text{Rangers}, \text{Choppers}, 4, 2>, <\text{Choppers}, \text{Ducks}, 0, 5>\}$
Relation schemas vs instances

- **Schema**: definition of the structure of the relation. Example:
  Teams have 3 attributes: name, home field, coach. No two teams can have the same name.

- **Notation for expressing a relation’s schema**
  \( \text{Teams}(\text{Name}, \text{HomeField}, \text{Coach}) \)

- **Instance**: particular data in the relation.
- Instances change constantly; schemas rarely.
- Conventional databases store the current version of the data. Databases that record the history are called *temporal* databases.
**Terminology**

- **relation** (table)
- **attribute** (column)
  Optionally, we can specify that attributes have domains; like types in a programming language
- **tuple** (row)
- **arity** of a relation: number of attributes (columns)
- **cardinality** of a relation: number of tuples (rows)

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<td>Choppers</td>
<td>High Park</td>
<td>Tom Cole</td>
<td></td>
</tr>
<tr>
<td>Crullers</td>
<td>WTCS</td>
<td>Anna Liu</td>
<td></td>
</tr>
</tbody>
</table>
A relation is a *set* of tuples, which means:
- there can be no duplicate tuples
- order of the tuples doesn't matter

In another model, relations are bags — a generalization of sets that allows duplicates.
Commercial DBMSs use this model.
But for now, we will stick with relations as sets.
Database schemas and instances

- **Database schema**: a set of relation schemas
- **Database instance**: a set of relation instances
Superkeys

- Superkey: a set of one or more attributes whose combined values are unique:
  - I.e., no two tuples can have the same values on all of these attributes.

- Formally:
  If attributes $a_1, a_2, \ldots, a_n$ form a superkey for relation $R$, $\not\exists$ tuples $t_1$ and $t_2$ such that
  
  $$(t_1.a_1 = t_2.a_1) \land (t_1.a_2 = t_2.a_2) \land \ldots \land (t_1.a_n = t_2.a_n)$$
Superkeys

- Example:
  - A relation called Course, with attributes department code, course number, and course name.
  - One tuple might be <“csc”, “343”, “Introduction to Databases”>
  - If department code + course number is a superkey for this relation, what is not allowed?

- Does every relation have a superkey?
- Can a relation have more than one superkey?
Keys

- A superkey may not be minimal.
  - I.e., you may be able to remove an attribute, and still have a set of attributes whose combined values are unique.

- **key**: a minimal superkey.

- We underline attributes in the schema to indicate that they form a key.
  
  Teams(Name, HomeField, Coach)

- Aside: Called “superkey” because it is a superset of some key.
  (Not necessarily a proper superset.)

- Can a relation have more than one key?
If a set of attributes is a key for a relation:

– It does not mean merely that there are no duplicates in a particular instance of the relation
– It means that in principle there cannot be any.
– Only a domain expert can determine that.

Often we invent an attribute to ensure all tuples will be unique. This predates databases. E.g., SIN, ISBN number.

A key is a kind of integrity constraint.
Example: Movies schema
References between relations

- Relations often refer to each other.
- Example:
  In the Roles relation, the tuple about Han Solo needs to say he is played by Ford.
- Rather than repeat information already in the Artists table, we store Ford’s key.
- If aID is a key for Artists, does that mean a particular aID can appear only once in Roles??
Foreign keys

- The referring attribute is called a **foreign key** because it refers to an attribute that is a key in another table.
- This gives us a way to refer to a single tuple in that relation.
- A foreign key may need to have several attributes.
Declaring foreign keys

- A bit of notation: $R[A]$
  - $R$ is a relation and
    $A$ is a list of attributes in $R$.
  - $R[A]$ is the set of all tuples from $R$, but with only the attributes in list $A$.

- We declare foreign key constraints this way: $R_1[X] \subseteq R_2[Y]$

- Example: Roles[aID] $\subseteq$ Movies[mID]
Foreign keys in the Movies schema
Referential integrity constraints

- These $R_1[X] \subseteq R_2[Y]$ relationships are called referential integrity constraints or inclusion dependencies.
- Not all referential integrity constraints are foreign key constraints.
- For example, we could say $\text{Artists[aID]} \subseteq \text{Roles[aID]}$
- In these cases, we are not referring to a unique tuple.
- $R_1[X] \subseteq R_2[Y]$ is a foreign key constraint iff $Y$ is a key for relation $R_2$. 
Designing a schema

- Mapping from the real world to a relational schema is surprisingly challenging and interesting.
- There are always many possible schemas.
- Two important goals:
  - Represent the data well. For example, avoid constraints that prevent expressing things that occur in the domain.
  - Avoid redundancy.
- Later, we’ll learn some elegant theory that provides sound principles for good design.
What’s next

- We will learn how to use SQL to
  - define a database’s structure,
  - put data in it, and
  - write queries on it.
- First we’ll learn how to write queries in relational algebra.
  - Relational algebra is the foundation for SQL.
  - Other important concepts, like query optimization, are defined in terms of RA.