In-class Exercises: BCNF

1. FD recap.
   
   (a) Create an instance of relation \( R(A, B, C, D, E) \) that violates this functional dependency: \( ABC \rightarrow DE \).

   \[
   \begin{array}{cccccc}
   A & B & C & D & E \\
   \hline
   8 & 2 & 3 & 1 & 6 \\
   8 & 2 & 3 & 1 & 6 \\
   8 & 4 & 1 & 5
   \end{array}
   \]

   all 3 are equal \( \Rightarrow \) so the D's and E's must be equal.
   But the E's are not!

   (b) Suppose we have a relation \( R(A, B, C, D, E) \). Does the instance below violate the functional dependency \( DB \rightarrow A \)?

   \[
   \begin{array}{cccccc}
   A & B & C & D & E \\
   \hline
   5 & 3 & 2 & 1 & 6 \\
   5 & 3 & 3 & 1 & 2 \\
   5 & 8 & 4 & 1 & 5
   \end{array}
   \]

   \( \Rightarrow \) this could be 99 and the FD would still not be violated.

2. Is a relation in BCNF?

   (a) Suppose we have a relation Students(SID, email, course, term, prof), and that these FDs hold:
   
   \( \{ \text{SID} \rightarrow \text{email}; \text{course, term} \rightarrow \text{prof}; \text{SID, course} \rightarrow \text{grade}. \} \). Is this relation in BCNF? No.

   \( \text{SID}^+ = \text{SID, email} \); \( \text{SID} \) is not a superkey in the relation.
   We know the relation is not in BCNF without looking at the other FDs.

   (b) Suppose we have a relation Customers(name, DOB, address, favouriteCar, manufacturer) and these FDs hold:
   
   \( \{ \text{name} \rightarrow \text{DOB}, \text{favouriteCar}; \text{favouriteCar} \rightarrow \text{manufacturer} \}. \) Is this relation in BCNF? No,

   \text{name}^+ \neq \text{name, DOB, favourite car, manufacturer}.

   This is not quite a key— it's lacking address.

   (c) Suppose we have a relation Parts(part, manufacturer, seller, price) and these FDs hold:
   
   \( \{ \text{part} \rightarrow \text{manufacturer}; \text{part, seller} \rightarrow \text{price} \}. \) Is this relation in BCNF? No.

   \text{part}^+ = \text{part, manufacturer}.

   (d) Suppose we have a relation \( R(A, B, C, D, E) \) and these FDs hold:
   
   \( \{ B \rightarrow AC; \text{CB} \rightarrow E; A \rightarrow D \}. \) Is this relation in BCNF? No.

   \( B^+ = \text{BACED} \); \( B \) is a superkey.

   \( \text{CB}^+ = \text{CBED} \) — in fact, we don't need the closure to know
   it's a superkey b/c it is a superset of key B.

   \( A^+ = AD \); \( A \) is not a superkey.
3. How does BCNF help? Consider again the relation relation Parts\{part, manufacturer, seller, price\} with these FDs:
\{ part \rightarrow manufacturer; \ part, seller \rightarrow price \}.

(a) Keeping in mind the FDs, make an instance of this relation that has redundant information.

\[
\begin{array}{ccc}
\text{part} & \text{manufacturer} & \text{seller} \\
A & \text{JHW} & \text{Rona} & 1.49 \\
A & \text{JHW} & \text{Walmart} & 1.48 \\
\end{array}
\]

Redundant! We could erase either one and know it must be JHW. NB: We need the FD to know this. Mere repetition does not imply redundancy.

(b) If we applied the decomposition step from BCNF decomposition, what attributes would each of the new relations have?

\[
\begin{align*}
R_1 &= \text{part, manufacturer} \\
R_2 &= \text{part, seller, price}
\end{align*}
\]

It is a coincidence that these correspond exactly to the FDs. That is often not the case.

(c) Project the FDs onto each of the new relations

\[
\begin{array}{cccc}
P & \text{M} & \text{P} \rightarrow \text{M} & \text{P} \rightarrow \text{M} \\
\checkmark & \checkmark & \checkmark & \checkmark \\
\end{array}
\]

\[
\begin{array}{cccc}
P & \text{S} & \text{Price} & \text{P} \rightarrow \text{M} \\
\checkmark & \checkmark & \checkmark & \checkmark \\
\end{array}
\]

\[
\begin{array}{cccc}
P & \text{M} & \text{P} \rightarrow \text{M} & \text{P} \rightarrow \text{M} \\
\checkmark & \checkmark & \checkmark & \checkmark \\
\end{array}
\]

\[
\begin{array}{cccc}
P & \text{S} & \text{Price} & \text{P} \rightarrow \text{M} \\
\checkmark & \checkmark & \checkmark & \checkmark \\
\end{array}
\]

\[
\begin{array}{cccc}
P & \text{M} & \text{P} \rightarrow \text{M} & \text{P} \rightarrow \text{M} \\
\checkmark & \checkmark & \checkmark & \checkmark \\
\end{array}
\]

\[
\begin{array}{cccc}
P & \text{S} & \text{Price} & \text{P} \rightarrow \text{M} \\
\checkmark & \checkmark & \checkmark & \checkmark \\
\end{array}
\]

\[
\begin{array}{cccc}
P & \text{M} & \text{P} \rightarrow \text{M} & \text{P} \rightarrow \text{M} \\
\checkmark & \checkmark & \checkmark & \checkmark \\
\end{array}
\]

1. Part \rightarrow Manufacturer is the only FD in R1

(d) Put the same data as in part (a) into your new schema. Is there any redundancy?

\[
\begin{array}{ccc}
\text{part} & \text{manufacturer} & \text{part} \\
A & \text{JHW} & A \\
\end{array}
\]

\[
\begin{array}{ccc}
\text{seller} & \text{price} & \text{seller} \\
A & \text{Rona} & 1.49 \\
A & \text{Walmart} & 1.48 \\
\end{array}
\]

\[
\begin{array}{ccc}
\text{part} & \text{seller} & \text{price} \\
A & \text{Rona} & 1.49 \\
A & \text{Walmart} & 1.48 \\
\end{array}
\]

No redundancy.

(e) Is it possible to create redundancy with this new schema?

No. A part may repeat as needed in R2, but that is not redundant. We "factored out" the fact about who manufactures what into R1, where we can state each such fact once.