In-class Exercises: Functional Dependencies

Suppose we have a relation $R$ with attributes $ABCD$

1. **What an FD means.** Suppose the functional dependency $BC \rightarrow D$ holds in $R$. Create an instance of $R$ that violates this FD.

2. **Equivalent sets of FDs.**
   
   (a) Are the sets $A \rightarrow BC$ and $A \rightarrow B, A \rightarrow C$ equivalent? If yes, explain why, intuitively. If no, construct an instance of $R$ that satisfies one set of FDs but not the other.

   (b) Are the sets $PQ \rightarrow R$ and $P \rightarrow Q, P \rightarrow R$ equivalent? If yes, explain why, intuitively. If no, construct an instance of $R$ that satisfies one set of FDs but not the other.
3. **Closure of attributes.** Suppose we have a relation on attributes $ABCDE$ with these FDs:

$$AC \rightarrow D, B \rightarrow E, DA \rightarrow B$$

(a) Find the closure of all attributes.

(b) Find the closure of all combinations of two attributes (pick only the interesting ones, whose closure isn’t just itself).

(c) Find the closure of $ABC$, $ACD$, and $BCD$.

4. **Does an FD follow from a set of FDs?** Suppose we have a relation on attributes $ABCDEF$ with these FDs:

$$AC \rightarrow F, CEF \rightarrow B, C \rightarrow D, DC \rightarrow A$$

(a) Does it follow that $C \rightarrow F$?

(b) Does it follow that $ACD \rightarrow B$?

5. **Equivalent sets of FDs - revisited.**

(a) Prove that the sets $A \rightarrow BC$ and $A \rightarrow B, A \rightarrow C$ are equivalent, using closures.