In-class Exercises: Functional Dependencies

Suppose we have a relation $R$ with attributes $ABCD$

1. **What an FD means.** Suppose the functional dependency $BC \rightarrow D$ holds in $R$. Create an instance of $R$ that violates this FD.

   **Solution:**

   In order to violate this FD, we need two tuples with the same value for $B$ and the same value for $C$ (both!), yet different values for $D$.

   $$
   \begin{array}{c|c|c|c}
   A & B & C & D \\
   \hline
   1 & 3 & 6 & 4 \\
   2 & 3 & 6 & 5 \\
   \end{array}
   $$

2. **Equivalent sets of FDs.**

   (a) Are the sets $A \rightarrow BC$ and $A \rightarrow B, A \rightarrow C$ equivalent? If yes, explain why. If your answer is no, construct an instance of $R$ that satisfies one set of FDs but not the other.

   **Solution:**

   These are equivalent — there is no instance of the relation that satisfies one but not the other. This can be proven, as follows:

   - Assume that $A \rightarrow BC$.
     - Under this assumption, $A^+ = ABC$.
     - Therefore $A \rightarrow B$, and $A \rightarrow C$.
   - Assume that $A \rightarrow B$, and $A \rightarrow C$.
     - Under this assumption, $A^+ = ABC$.
     - Therefore $A \rightarrow BC$.
   - Therefore each set of FDs follows from the other. They are equivalent.

   (b) Are the sets $PQ \rightarrow R$ and $P \rightarrow Q, P \rightarrow R$ equivalent? If yes, explain why. If no, construct an instance of $R$ that satisfies one set of FDs but not the other.

   **Solution:**

   These are not equivalent, as demonstrated by this instance that satisfies $PQ \rightarrow R$ but not $P \rightarrow Q, P \rightarrow R$:

   $$
   \begin{array}{c|c|c}
   P & Q & R \\
   \hline
   1 & 2 & 4 \\
   3 & 2 & 5 \\
   \end{array}
   $$

   In fact we can always “split the RHS” of an FD.

3. **Does an FD follow from a set of FDs?** Suppose we have a relation on attributes $ABCDEF$ with these FDs:

   $$
   AC \rightarrow F, \; CEF \rightarrow B, \; C \rightarrow D, \; DC \rightarrow A
   $$

   (a) Does it follow that $C \rightarrow F$?

   (b) Does it follow that $ACD \rightarrow B$?

   **Solution:**

   We use the closure test to check whether an FD follow from a set of FDs.

   $C^+ = CDAF$. Therefore, $C \rightarrow F$ does follow.

   $ACD^+ = ACDF$. Therefore, $ACD \rightarrow B$ does not follow.
4. **Projecting a set of FDs onto a subset of the attributes.** Suppose we have a relation on attributes \( ABCDE \) with these FDs:

\[
A \rightarrow C, \quad C \rightarrow E, \quad E \rightarrow BD
\]

Project the FDs onto attributes ABC:

**Solution:**

To project onto a set of attributes, we systematically consider every possible LHS of an FD that might hold on those attributes.

- \( A^+ = ACEBD \), therefore \( A \rightarrow BC \). (It also functionally determines \( DE \), but these are not in our set of attributes. And it functionally determines itself, but we don’t need to write down dependencies that are tautologies.)
- \( B^+ = B \). This yields no FDs for our set of attributes.
- \( C^+ = CEBD \), therefore \( C \rightarrow B \).
- We don’t need to consider any supersets of \( A \). \( A \) already determines all of our attributes \( ABC \), so supersets of \( A \) will be only yield FDs that already follow from \( A \rightarrow BC \).
- The only remaining subset of the attributes \( ABC \) to consider is \( BC \). \( BC^+ = BCED \). This yields no FDs for our set of attributes.
- So the projection of the FDs onto \( ABC \) is: \{ \( A \rightarrow BC \), \( C \rightarrow B \) \}.

5. **Minimal Basis of FDs.** Suppose we have a relation on attributes \( ABCDEG \) with these FDs:

\[
A \rightarrow B, \quad ABCD \rightarrow E, \quad G \rightarrow A, \quad G \rightarrow B
\]

Find the minimal basis (aka minimal cover) for these FDs:

**Solution:**

(Solution guide available in Week 10 slides for Section L5101.)