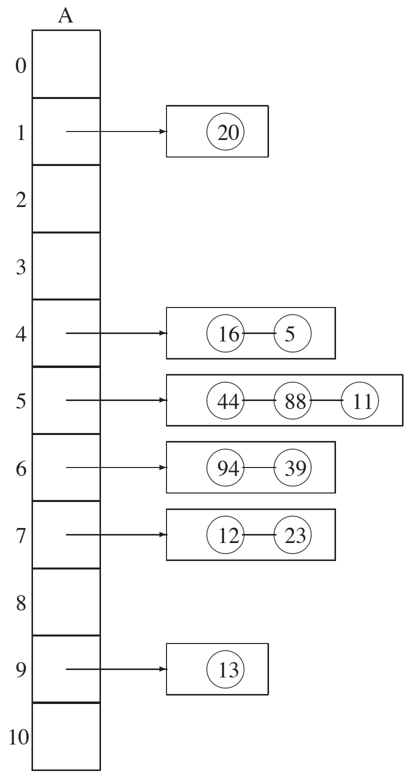
Tutorial – Hashing

1. (2.19) Draw the 11-item hash table resulting from hashing the keys 12,44,13,88,23,94,11,39,20,16,5 using the hash function h(i) = (2i+5)mod 11 and handling collisions using chaining.



2. (2.20) Redo with linear probing.

11	39	20	5	16	44	88	12	23	13	94

3. (2.21) Redo with quadratic probing

	20	16	11	39	44	88	12	23	13	94

4. (2.22) Redo with double hashing and the second hash function being h'(k) = 7-(kmod 7).

11	23	20	16	39	44	94	12	88	13	5

5. Consider a hashing scheme where each hash table location can hold up to n records and linear probing is used to resolve collisions. No deletions have occurred on the table.

   Let k_i be the key of the i^{th} record to be inserted into the hash table. Let

$$H_{j,p} = \{k_i | h(k_i) = j \text{ and } i < p\}.$$

   Consider the p^{th} record with key k_p and hash value $h(k_p) = j$. Assume that $|H_{j,p}| < n$, i.e., that less than n previously inserted keys hash to bucket j as their home bucket.

Is it possible to have a situation where the insertion of the p^{th} record requires a probe because bucket j is full?

Answer the question by considering the case when $p \leq 2n$ and the case when $p > 2n$. In each of these cases either give an example to show that this is possible or provide a proof that it can never occur.

Solution

- Consider $p > 2n$.

  Yes, it is possible. Bucket j could be filled from overflow from a previous bucket. Consider $n = 3$, $p = 7$, $H(k) = k$ mod 7. Insert the following keys in this order 8,15,22,29,3,10,17. Of the first n keys, only 3 and 10 hash to bucket 3. Yet, when 17 hashes to bucket 3, it finds it full and requires a probe to bucket 5.

- Now consider $p \leq 2n$.

  At most 2n-1 records are in the table during the insert of record p. Let a be the home bucket for record p. Assume that on the insertion of p, a probe is required. Therefore a holds n records. But at most n-1 records have had a as their home bucket so at least 1 record was placed in a whose home bucket is not a and is there because of a probe. The home bucket of this record must also be full. It must contain n records. This gives us at least 2n records in the table which is a contradiction. So this situation can not occur if $p \leq 2n$.