An Introduction to Heuristic Search-Based Planning

Rick Valenzano and Sheila McIlraith

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Lecture Plan

- Planning as pathfinding in a graph
  - Heuristic-based planning
- From Dijkstra's to Uniform-Cost Search
- Heuristics from abstraction and relaxation
- The A* Algorithm

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Quick Survey

- A*?
- IDA*?
- Weighted A*?
- Greedy Best-First Search?
- Enforced Hill-Climbing?
- A_{ϵ}^{*}? EES?

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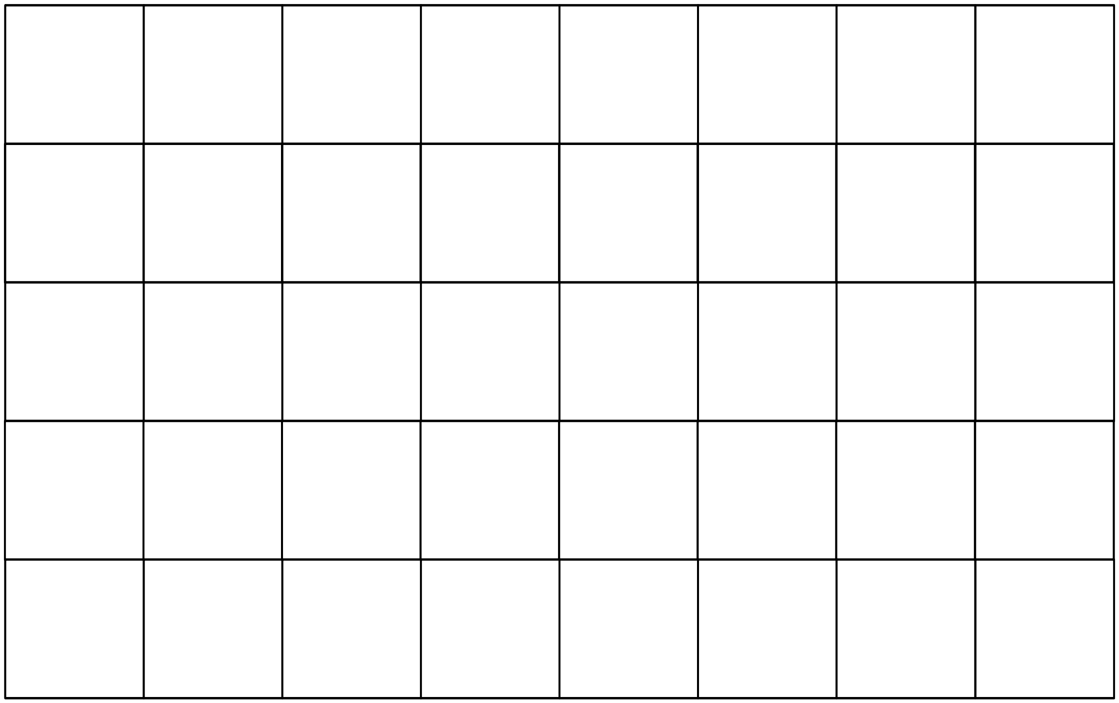
Floortile from IPC 2011



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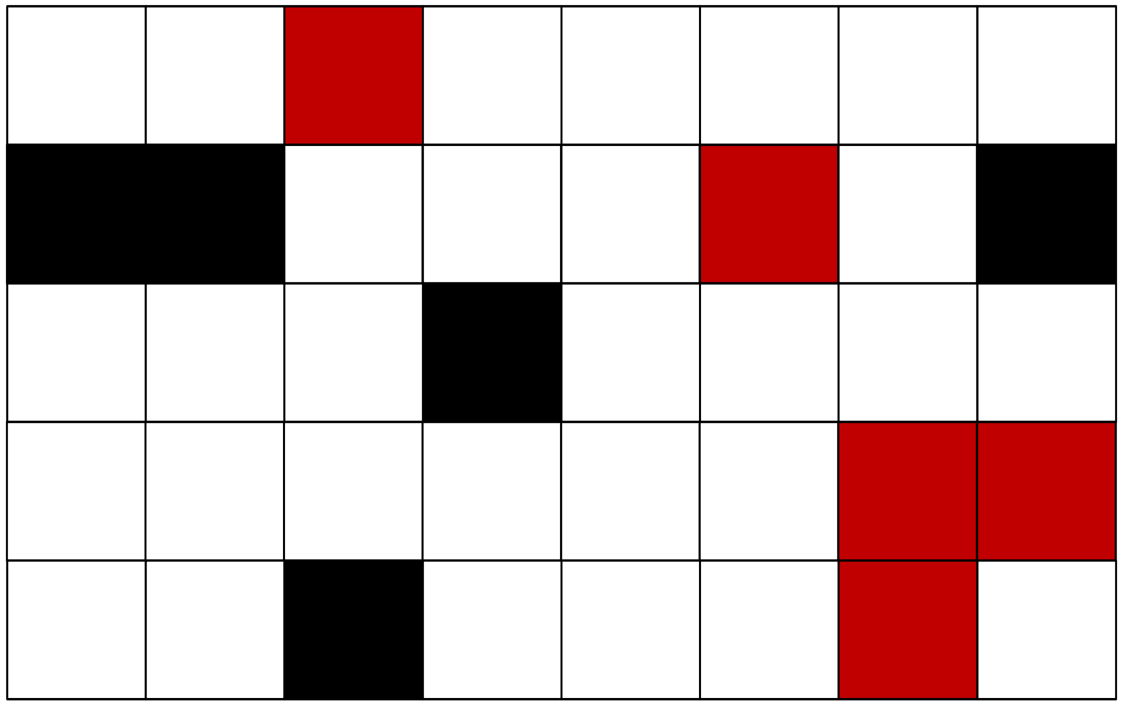

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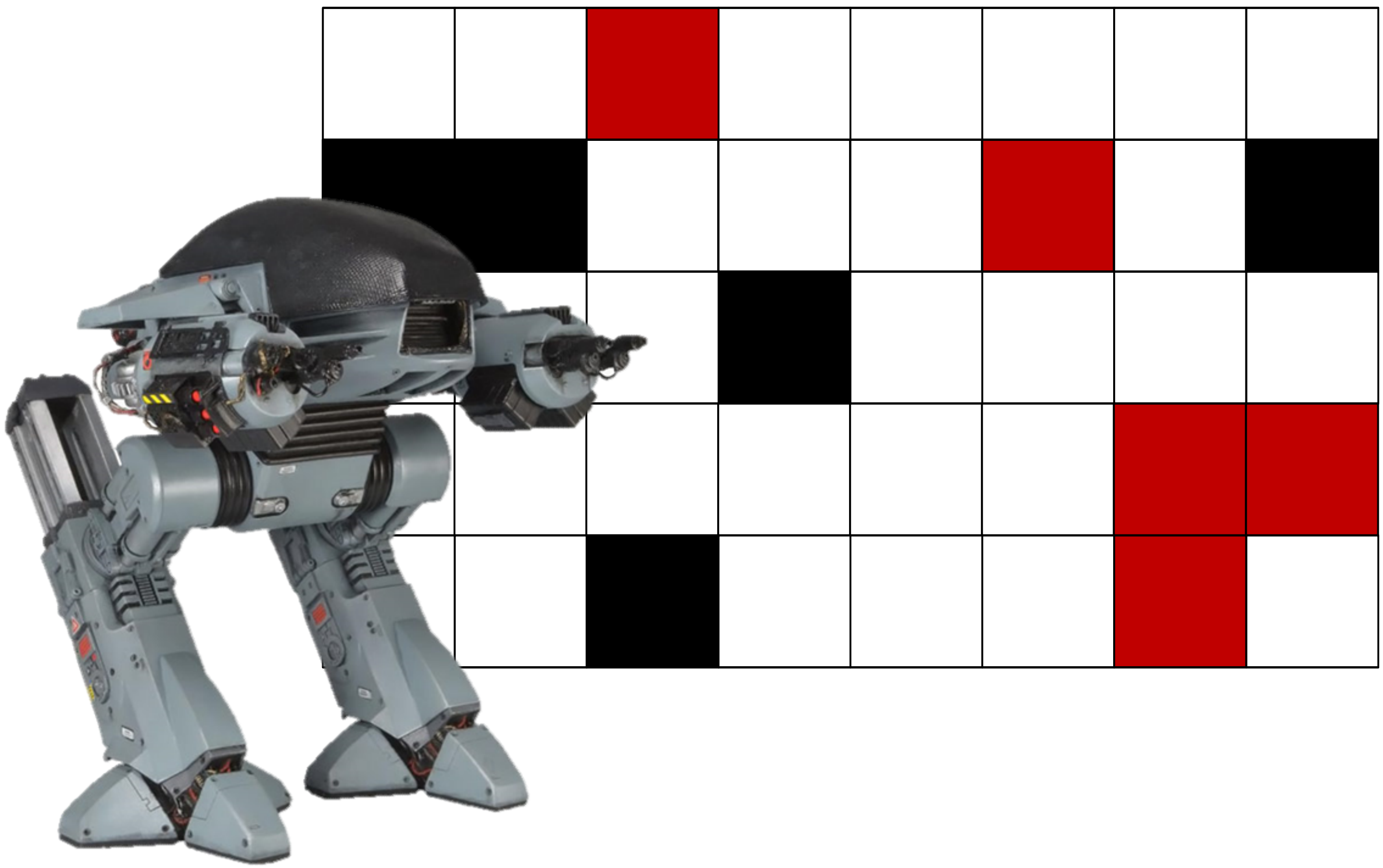

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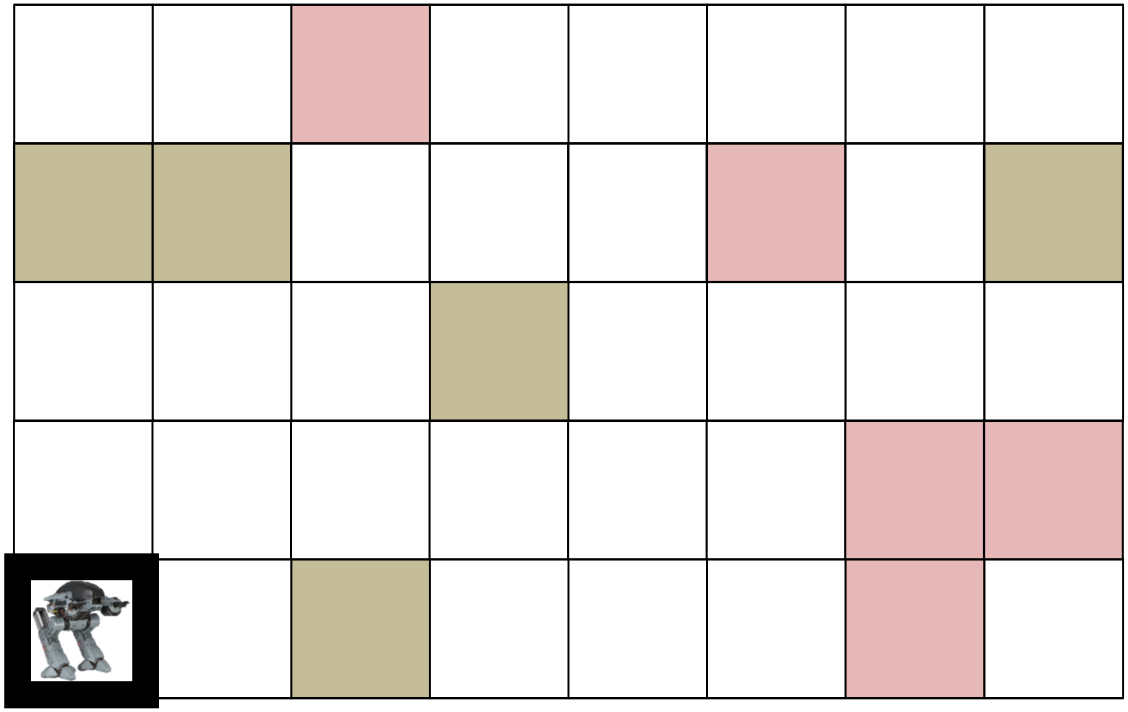

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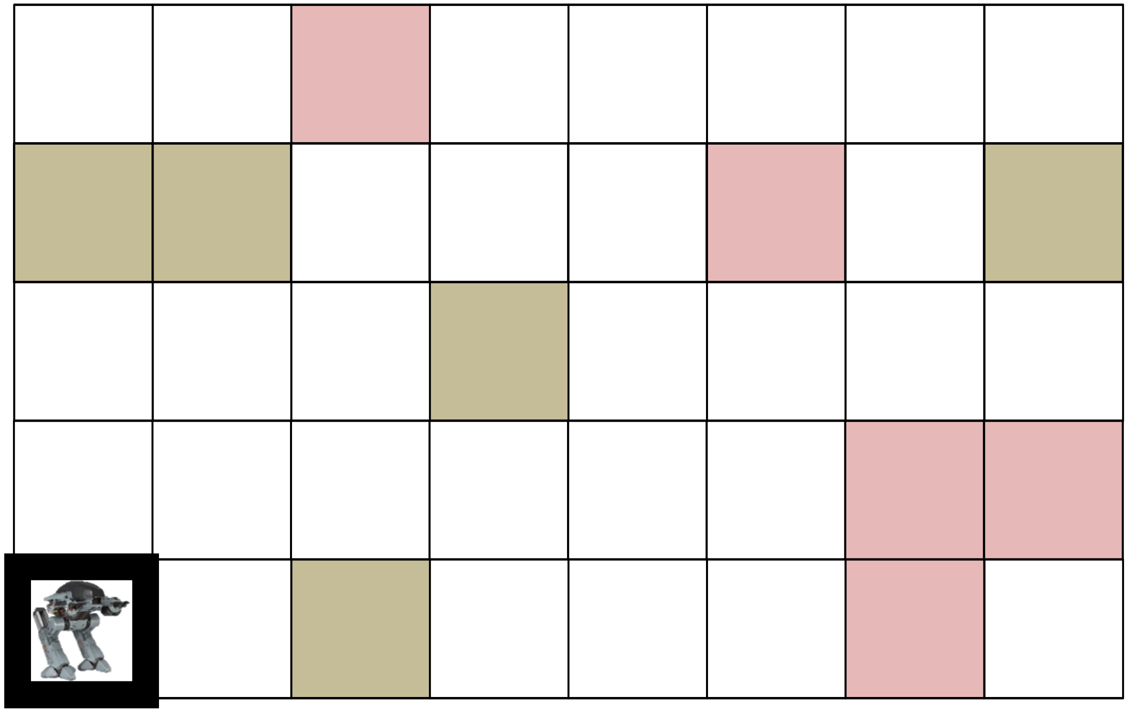
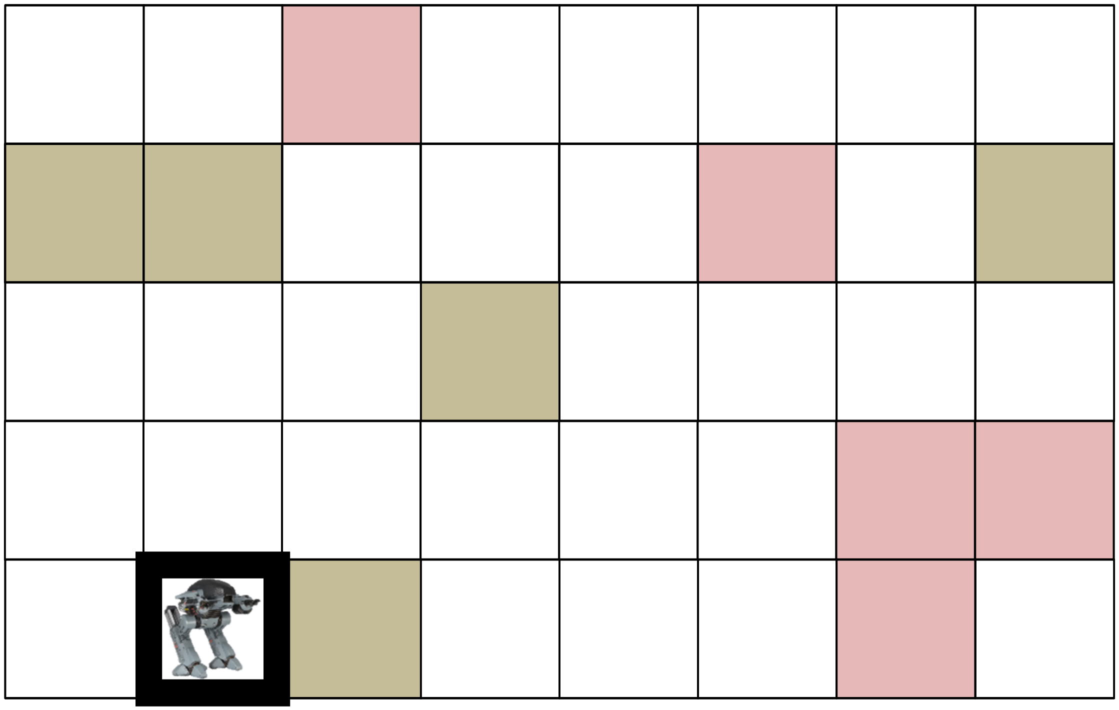


Move Action: MOVE-0-0-0-1

Pre: AT-0-0, WHITE-0-1

Post: AT-0-1, not(AT-0-0)

Floortile from IPC 2011



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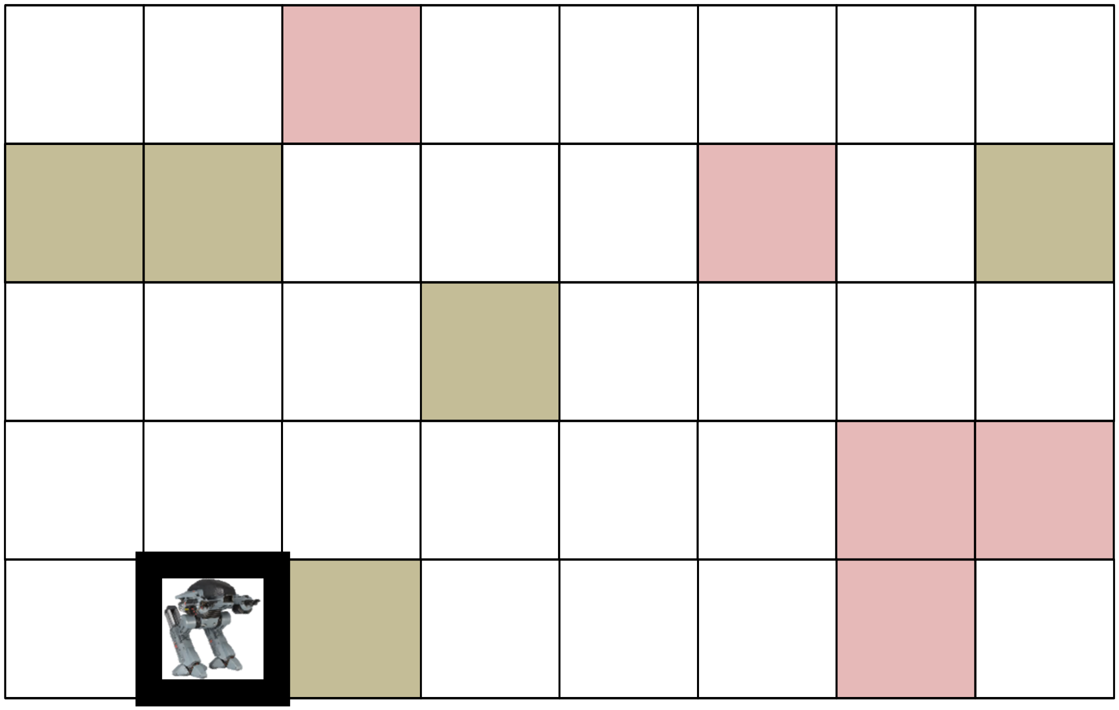

Paint Action: PAINT-B-0-1-0-2

Pre: AT-0-1, LOADED-B,
WHITE-0-2, NEED-B-0-2

Post: BLACK-0-2, not(WHITE-0-2)

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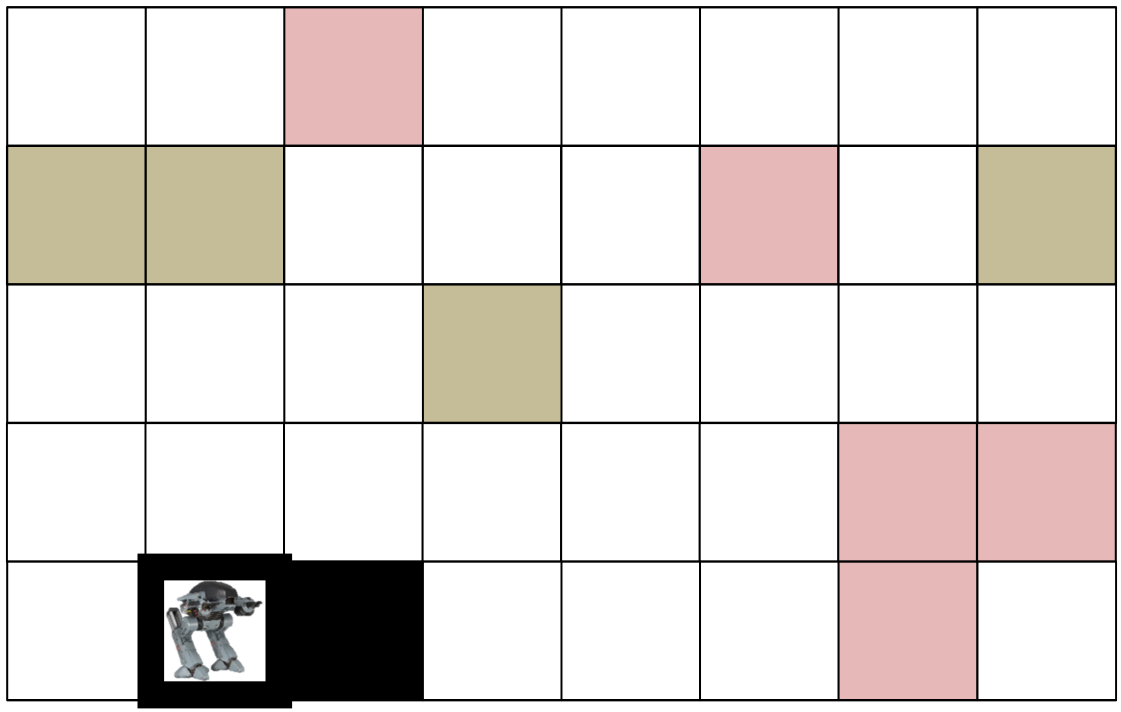
Floortile from IPC 2011



Paint Action: PAINT-B-0-1-0-2

Pre: AT-0-1, LOADED-B,
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Post: BLACK-0-2, not(WHITE-0-2)

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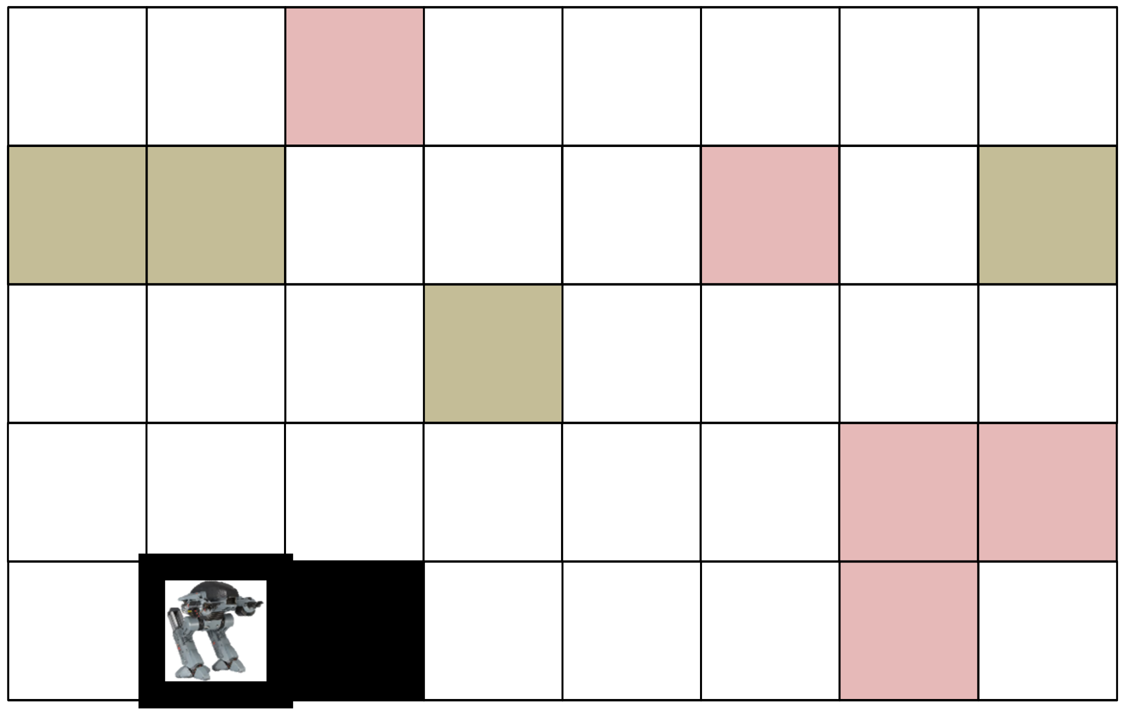
Floortile from IPC 2011



Load Action: LOAD-RED

Pre: LOADED-B

Post: LOADED-R, not(LOADED-B)

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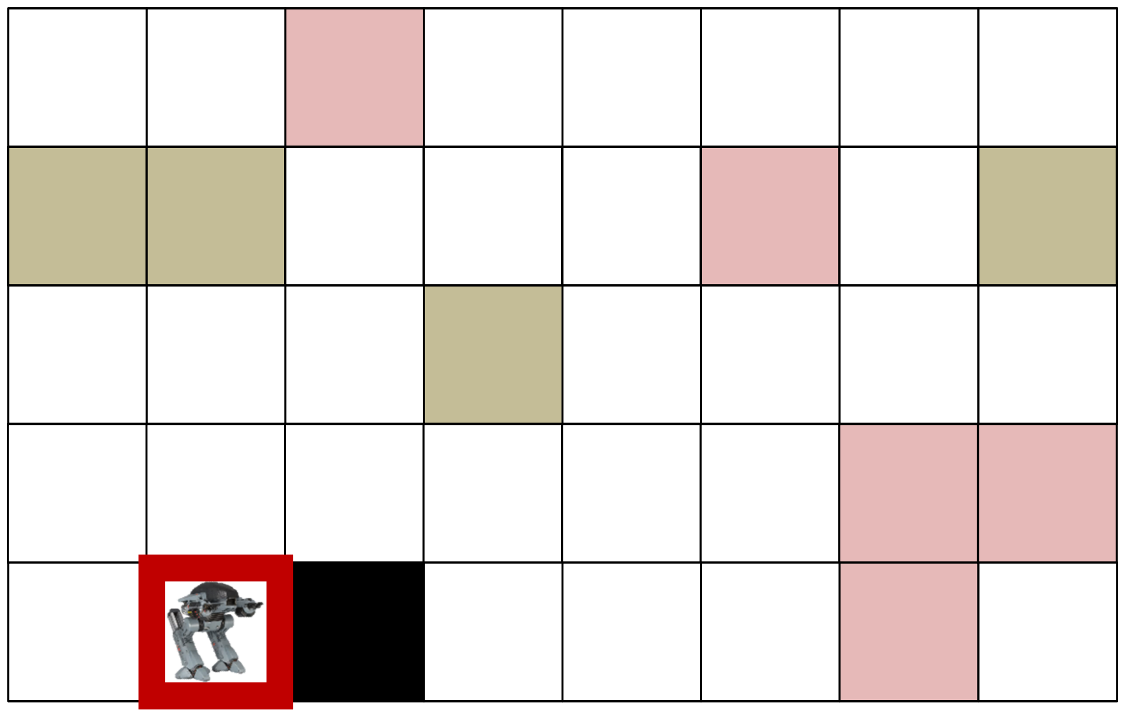
Floortile from IPC 2011



Load Action: LOAD-RED

Pre: LOADED-B

Post: LOADED-R, not(LOADED-B)

Floortile from IPC 2011

Initial State	Goal
AT-0-0	BLACK-0-2
LOADED-B	BLACK-3-0
WHITE-0-0	BLACK-3-1
WHITE-0-1	RED-4-2
WHITE-0-2	BLACK-2-3
NEED-B-0-2	RED-3-5
…	…

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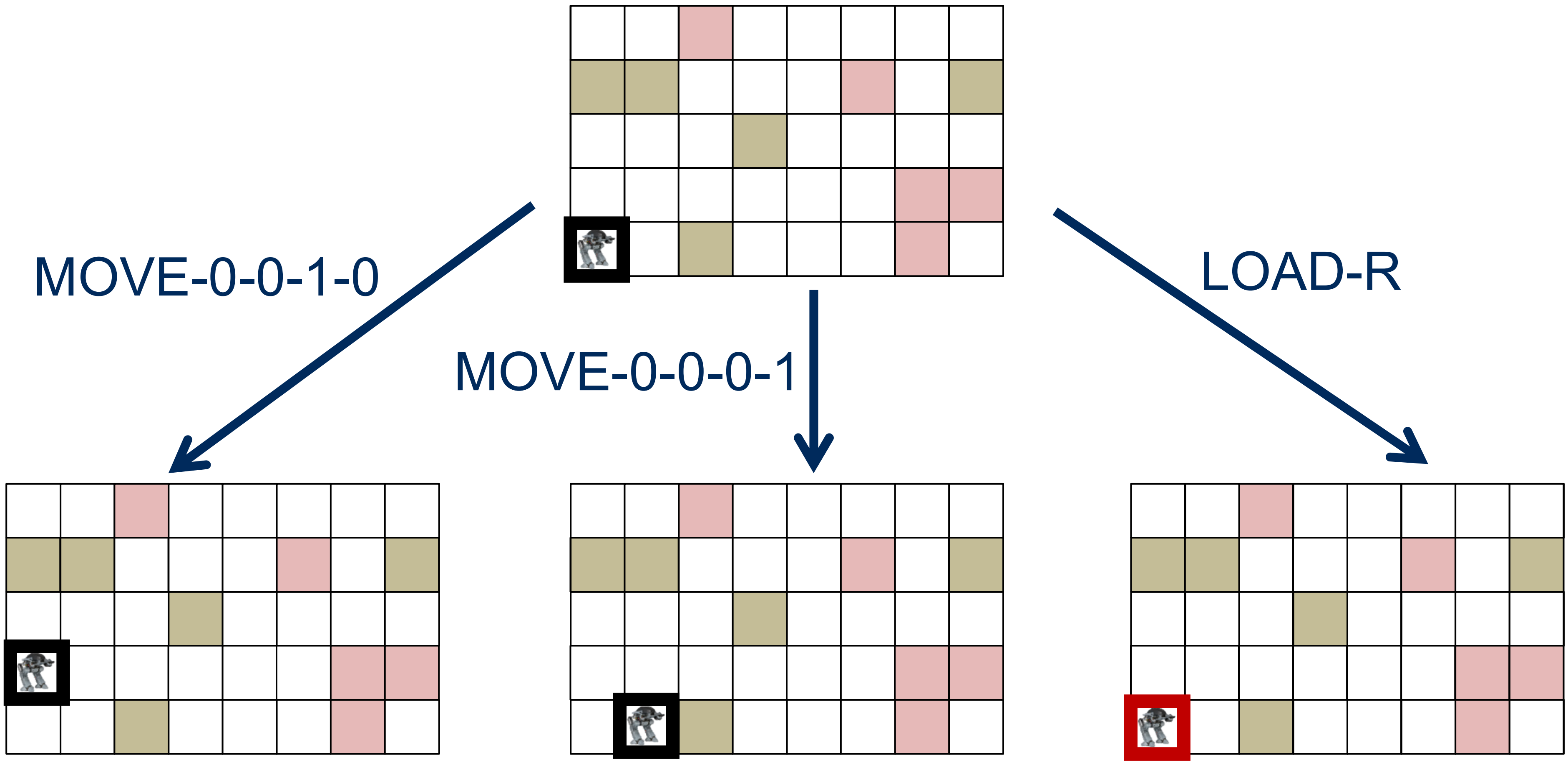
Floortile from IPC 2011



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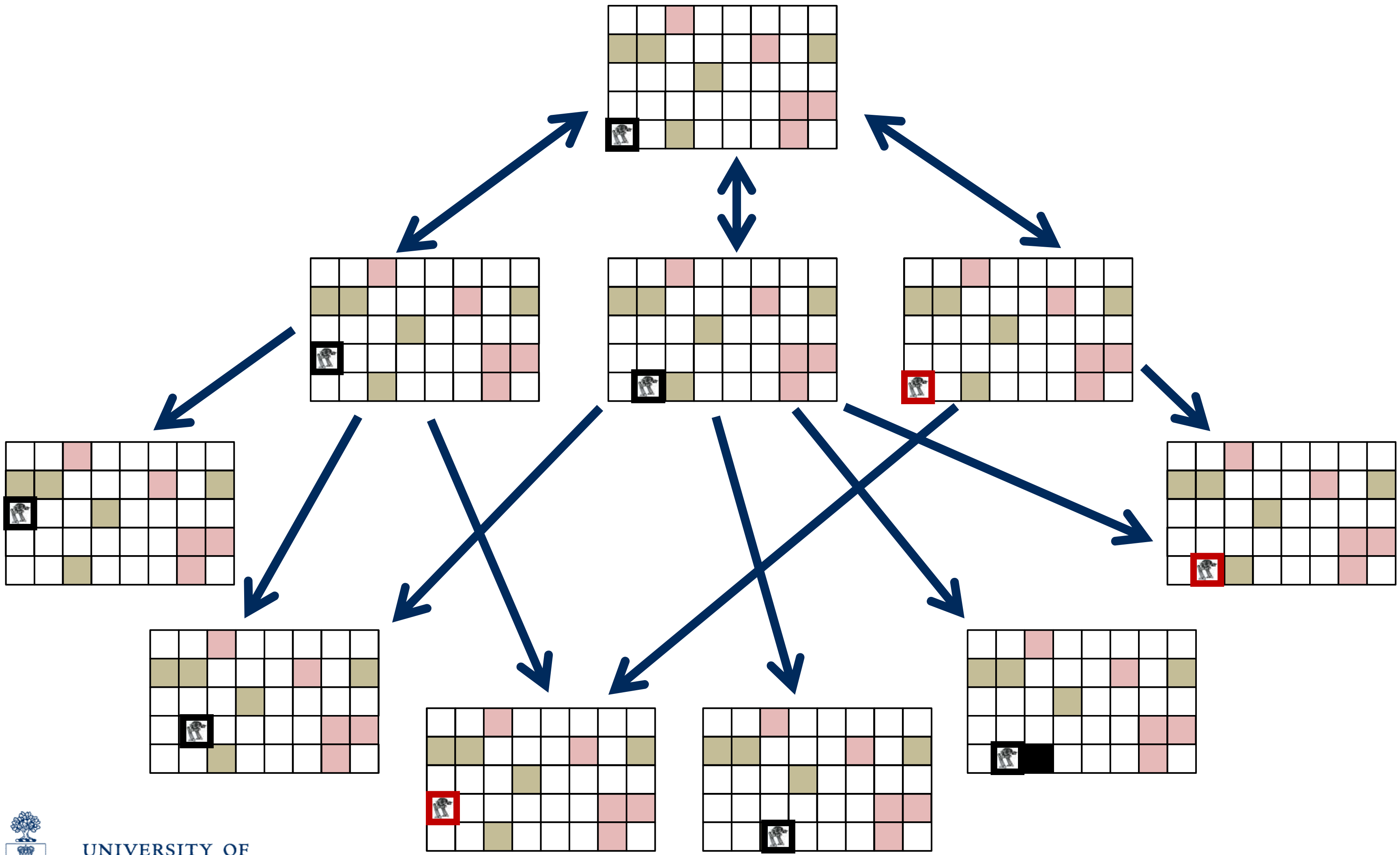

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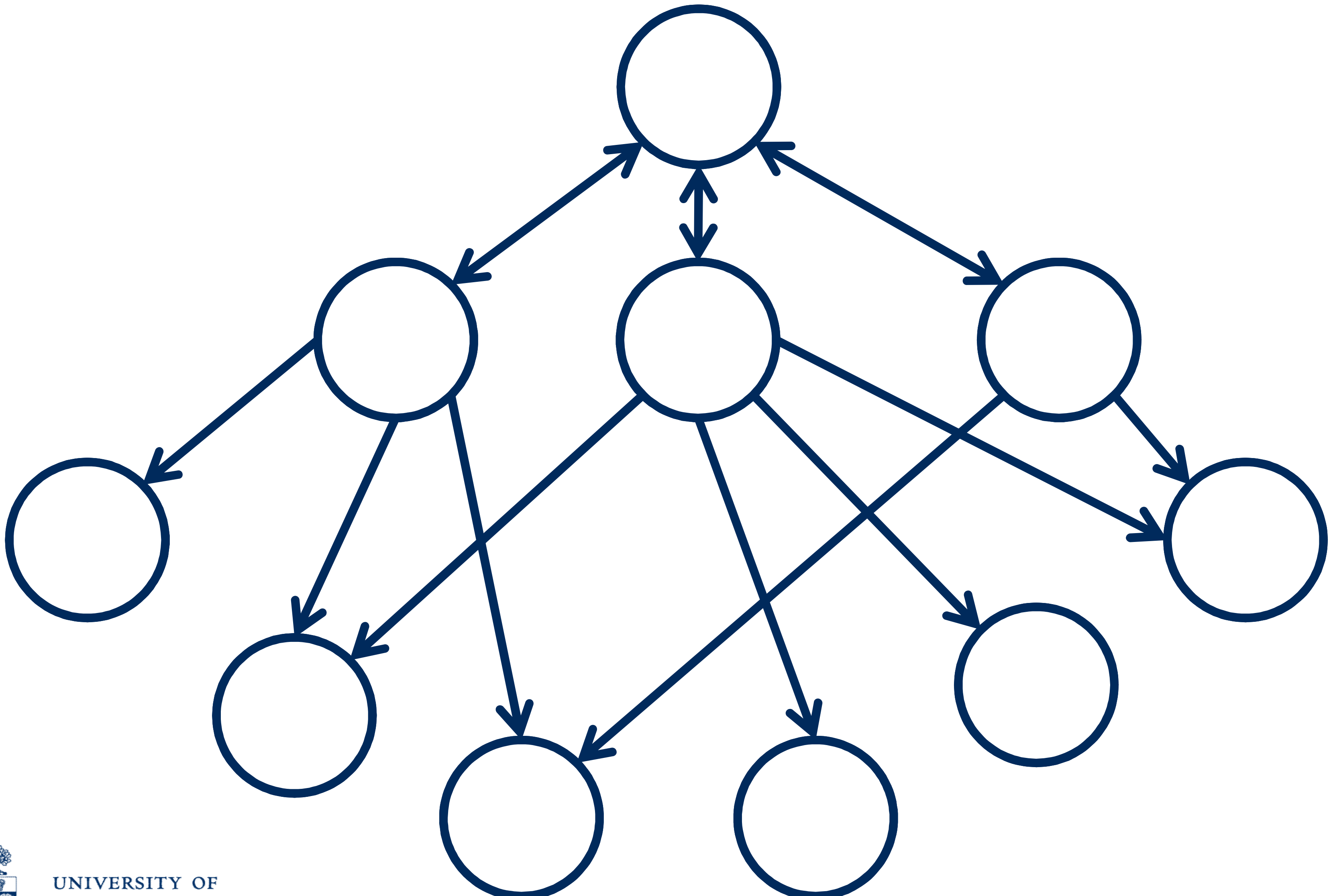
Graph Underlying Floortile



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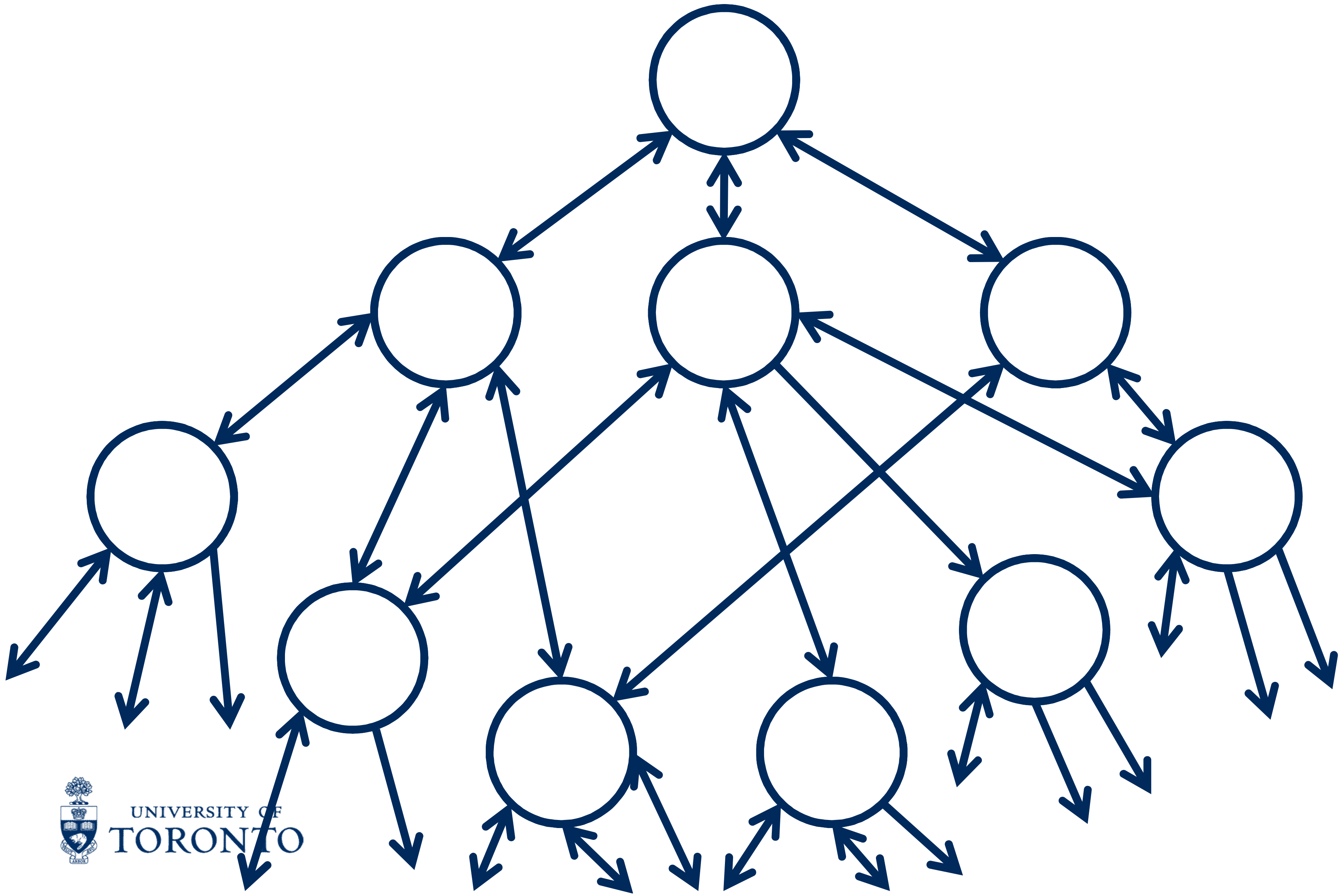
Graph Underlying Floortile



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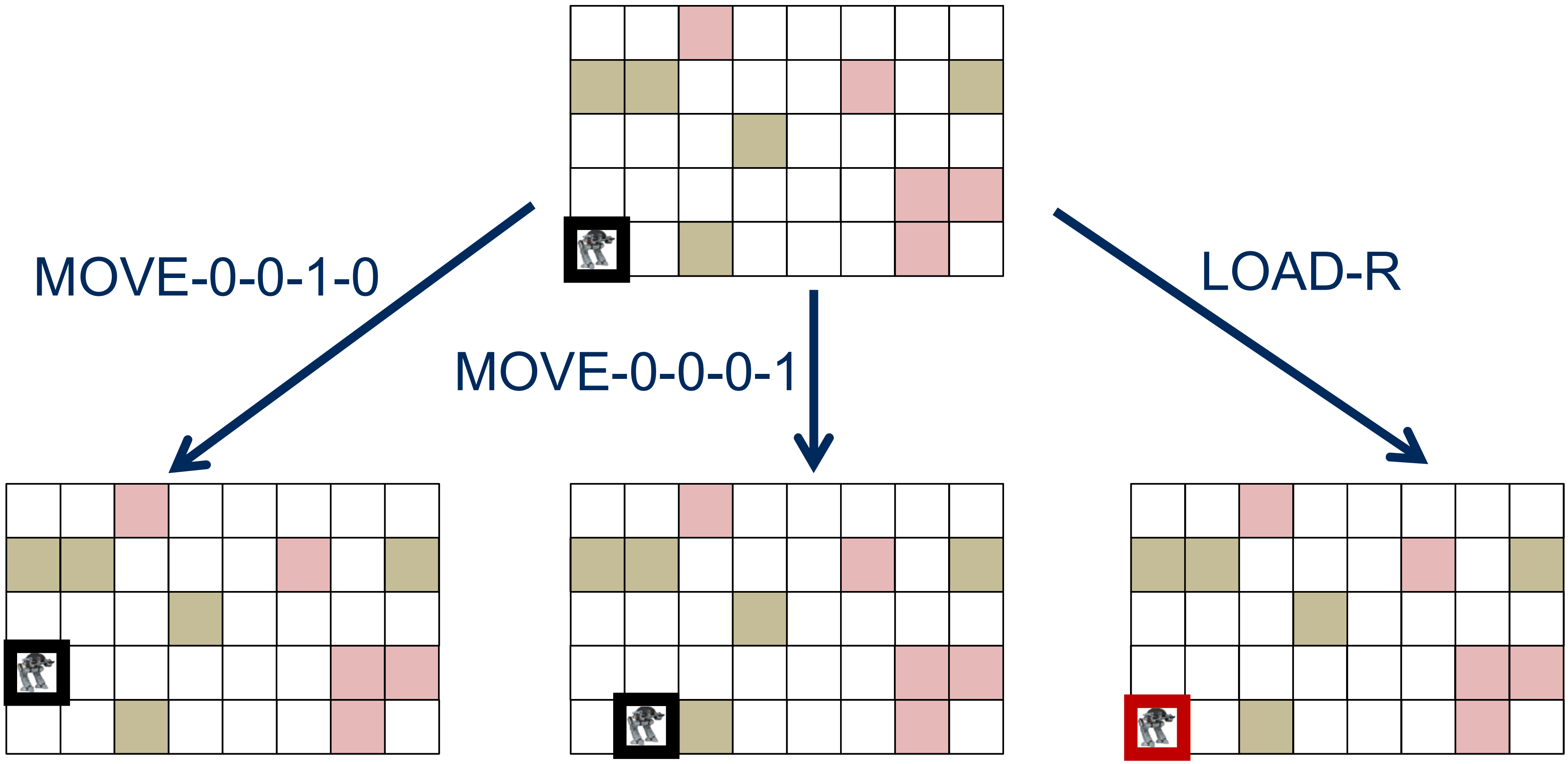
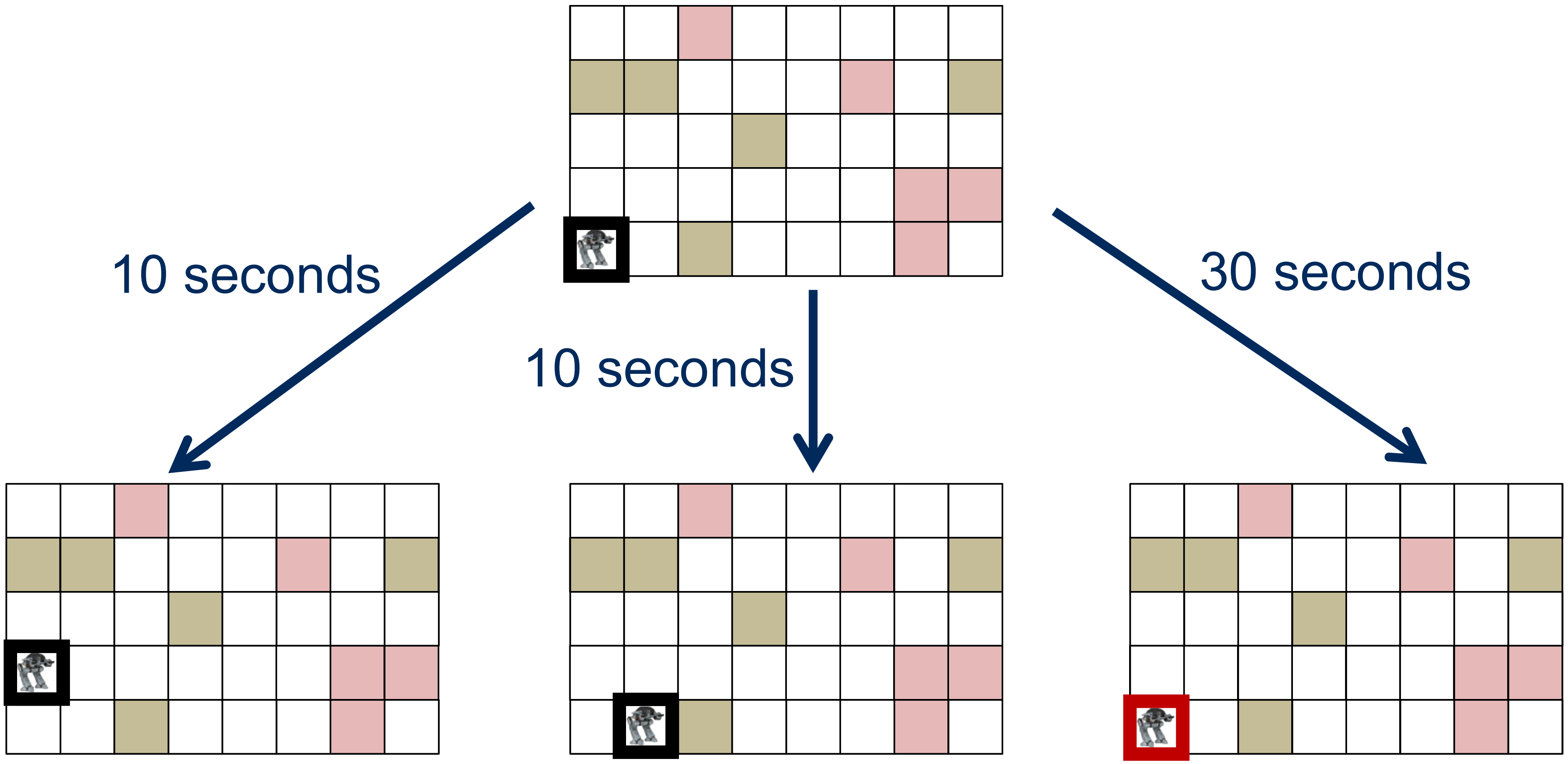
Action Costs



Action Costs



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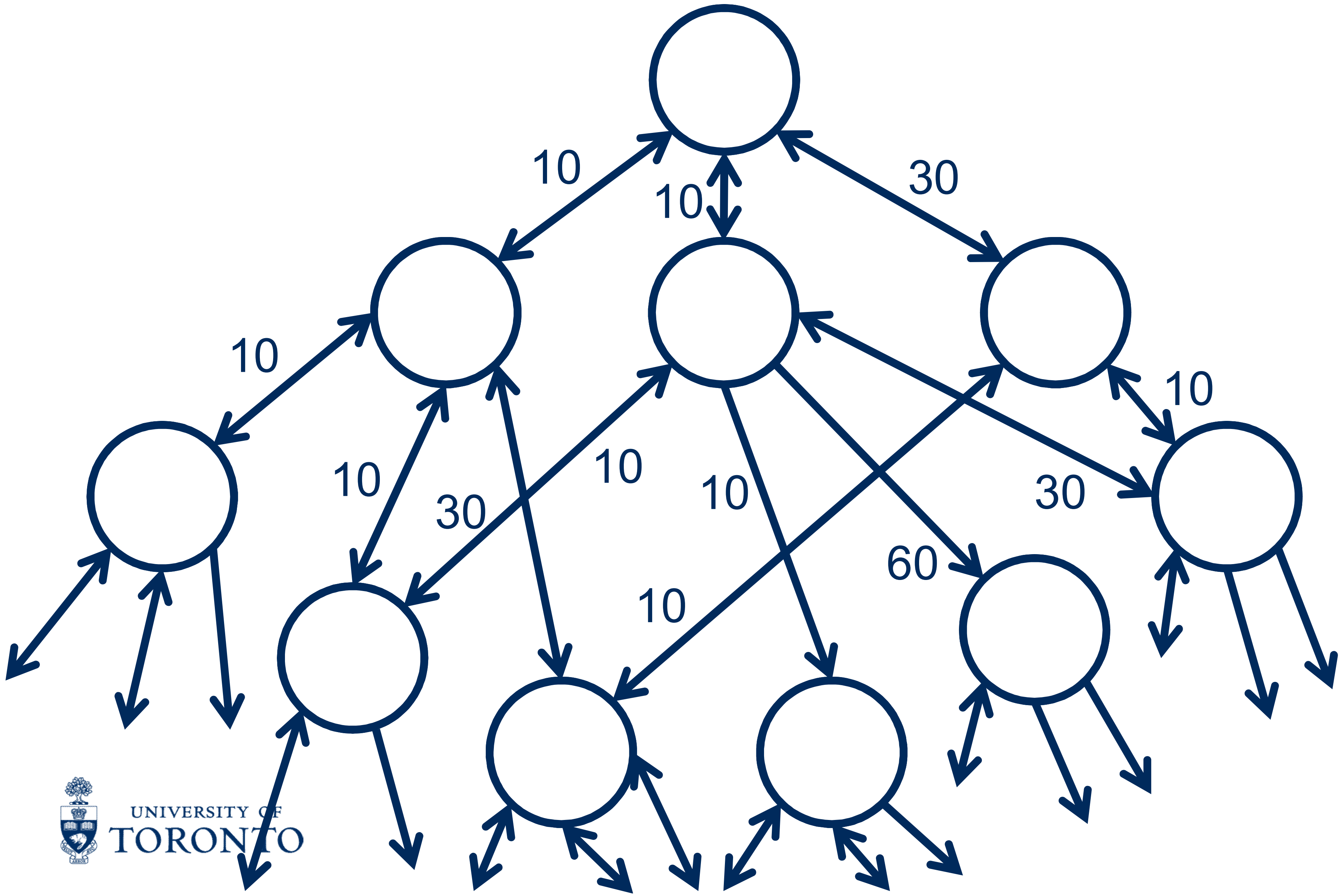
Edge Weights



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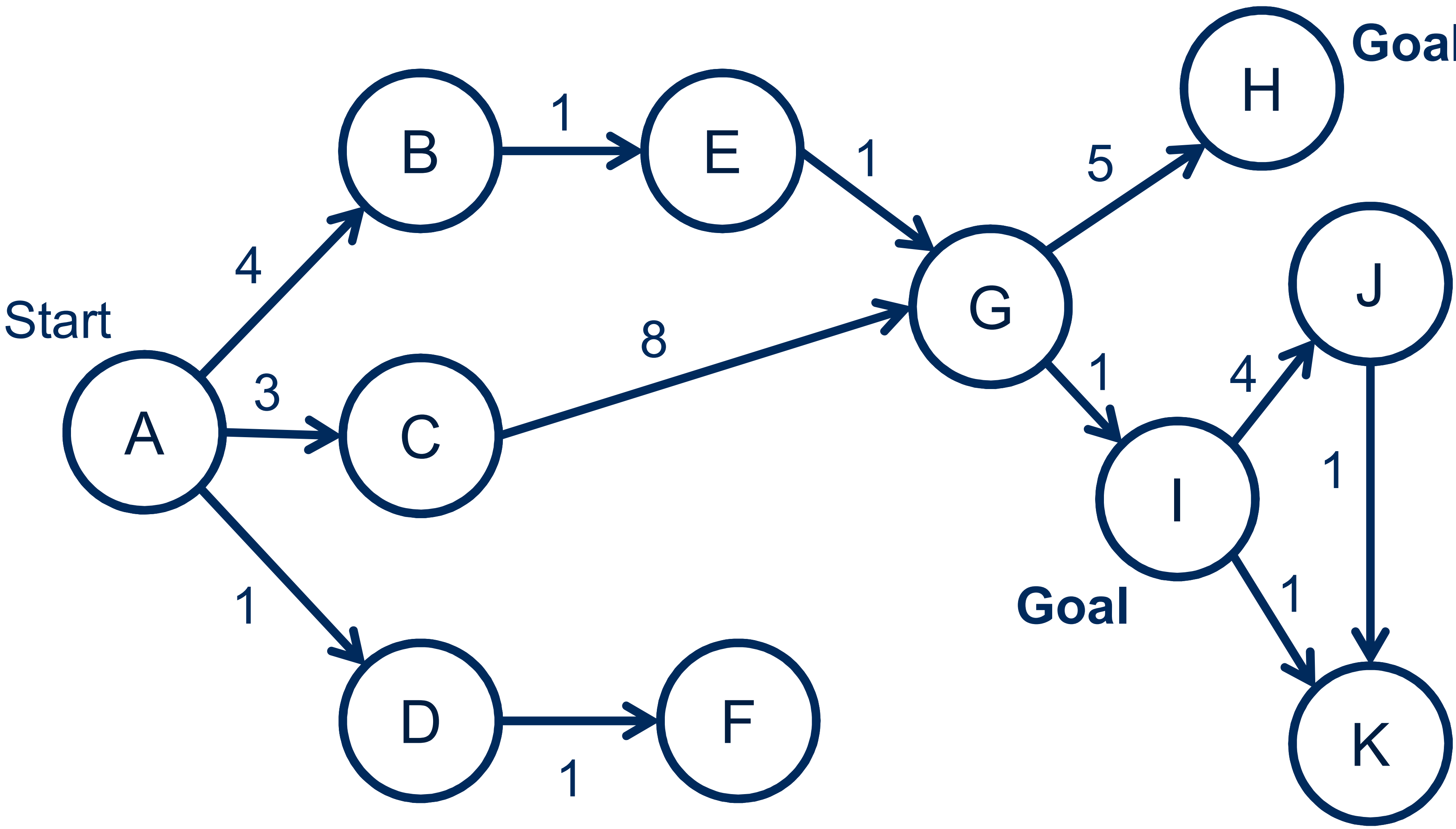
Planning as Graph-Search

- Can generate the underlying graph
- Use the goal test function to label goal nodes
- Use a standard graph-search algorithm

Dijkstra's Search



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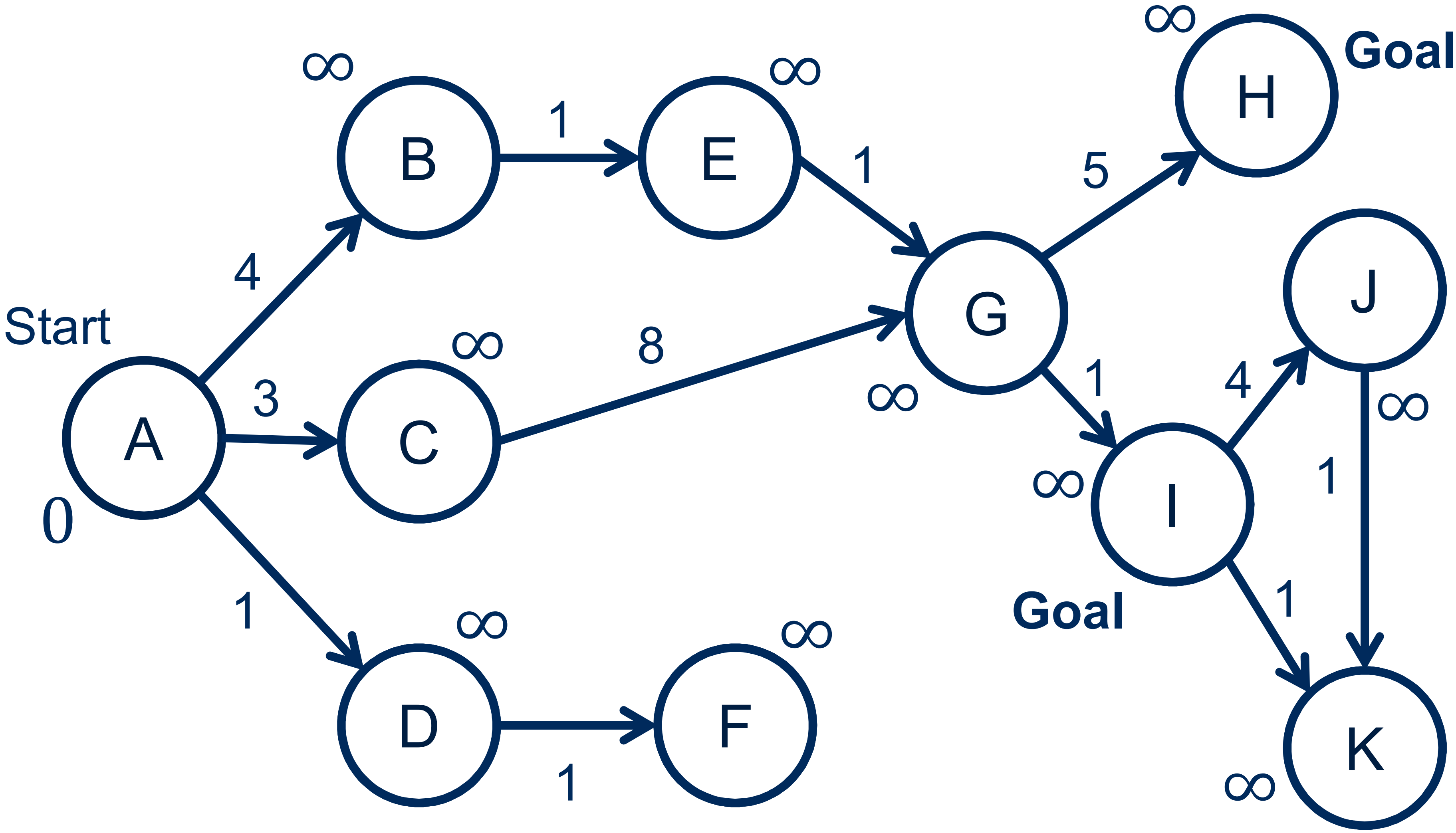
Dijkstra's Search



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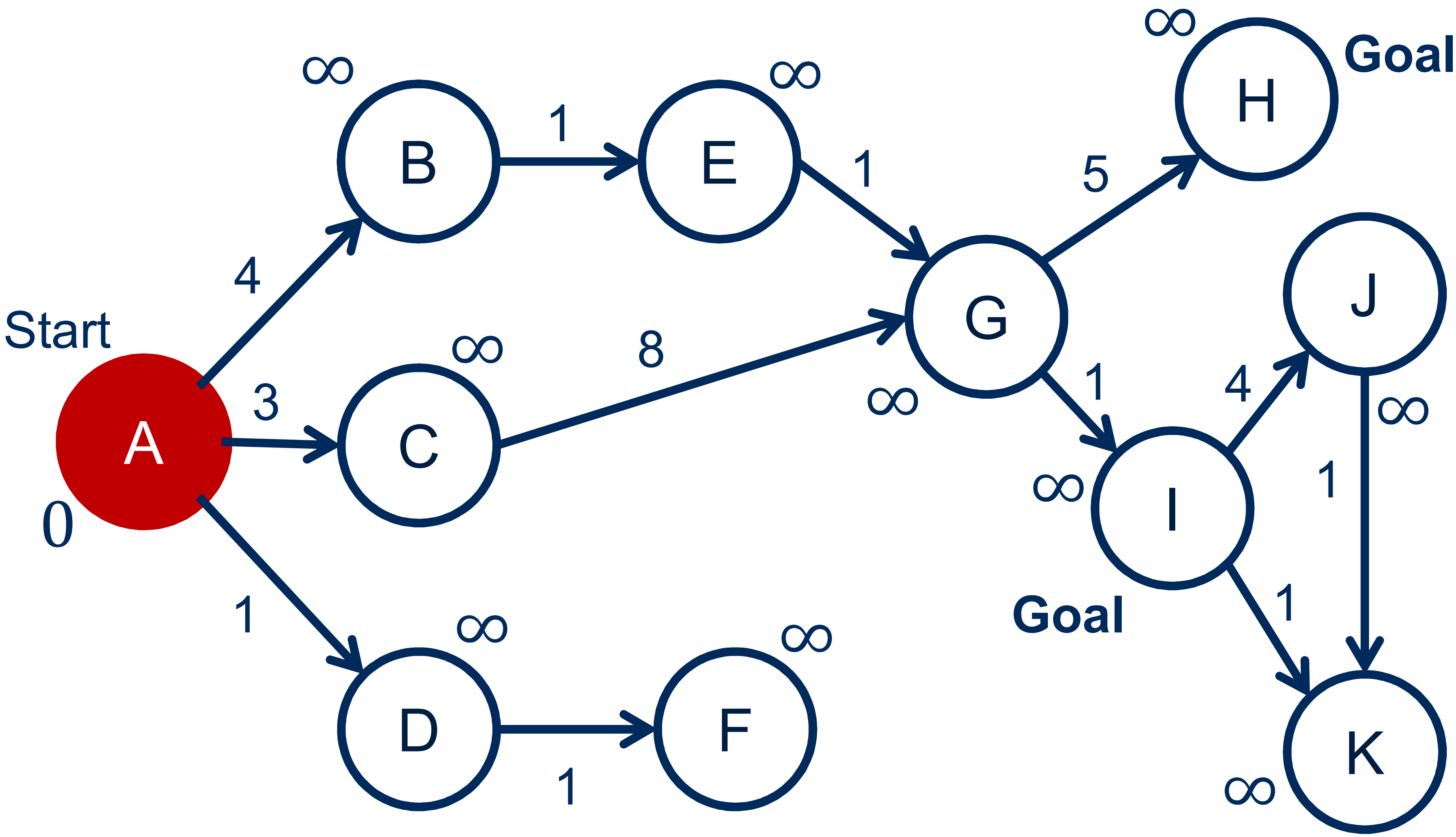

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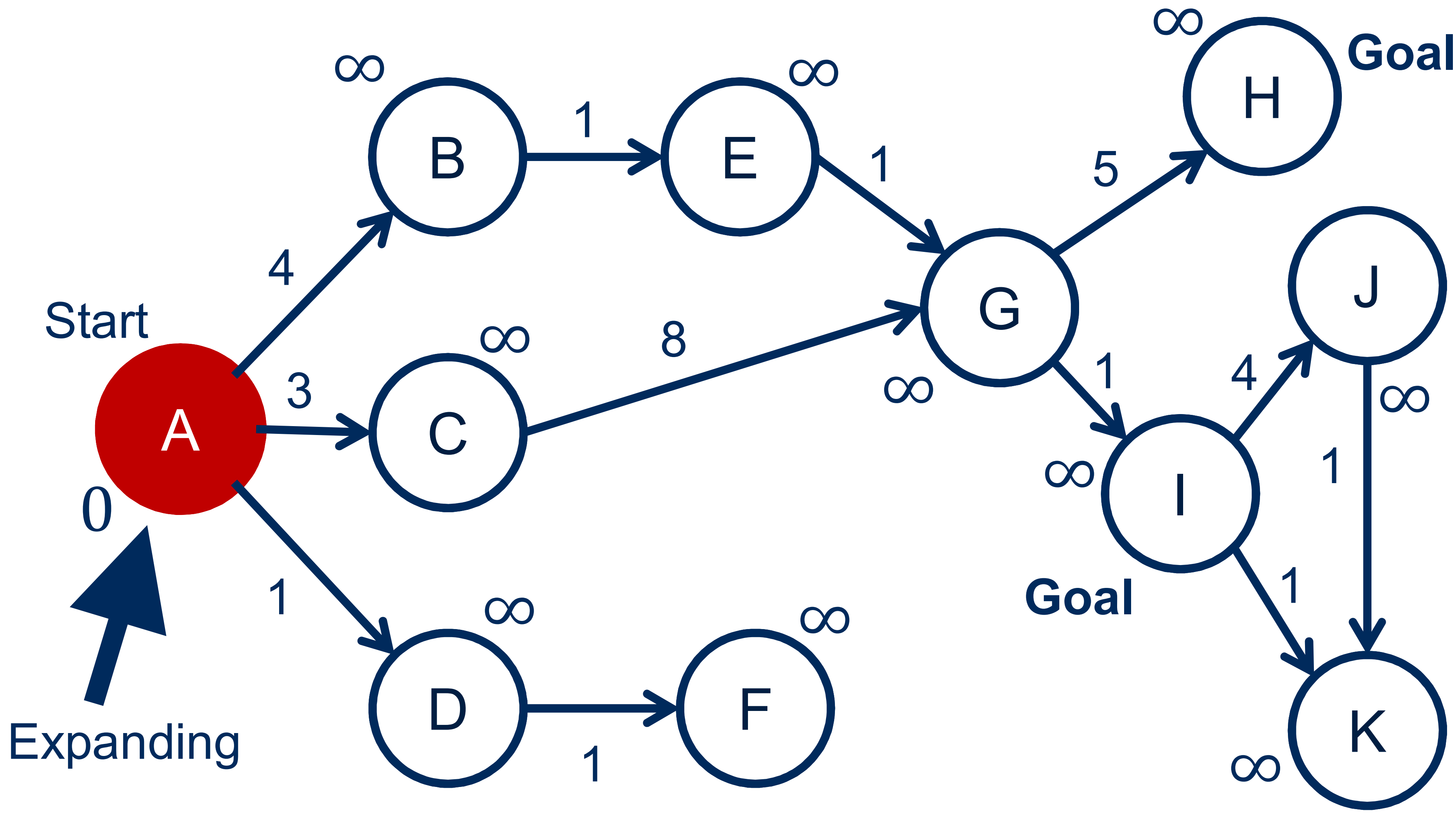

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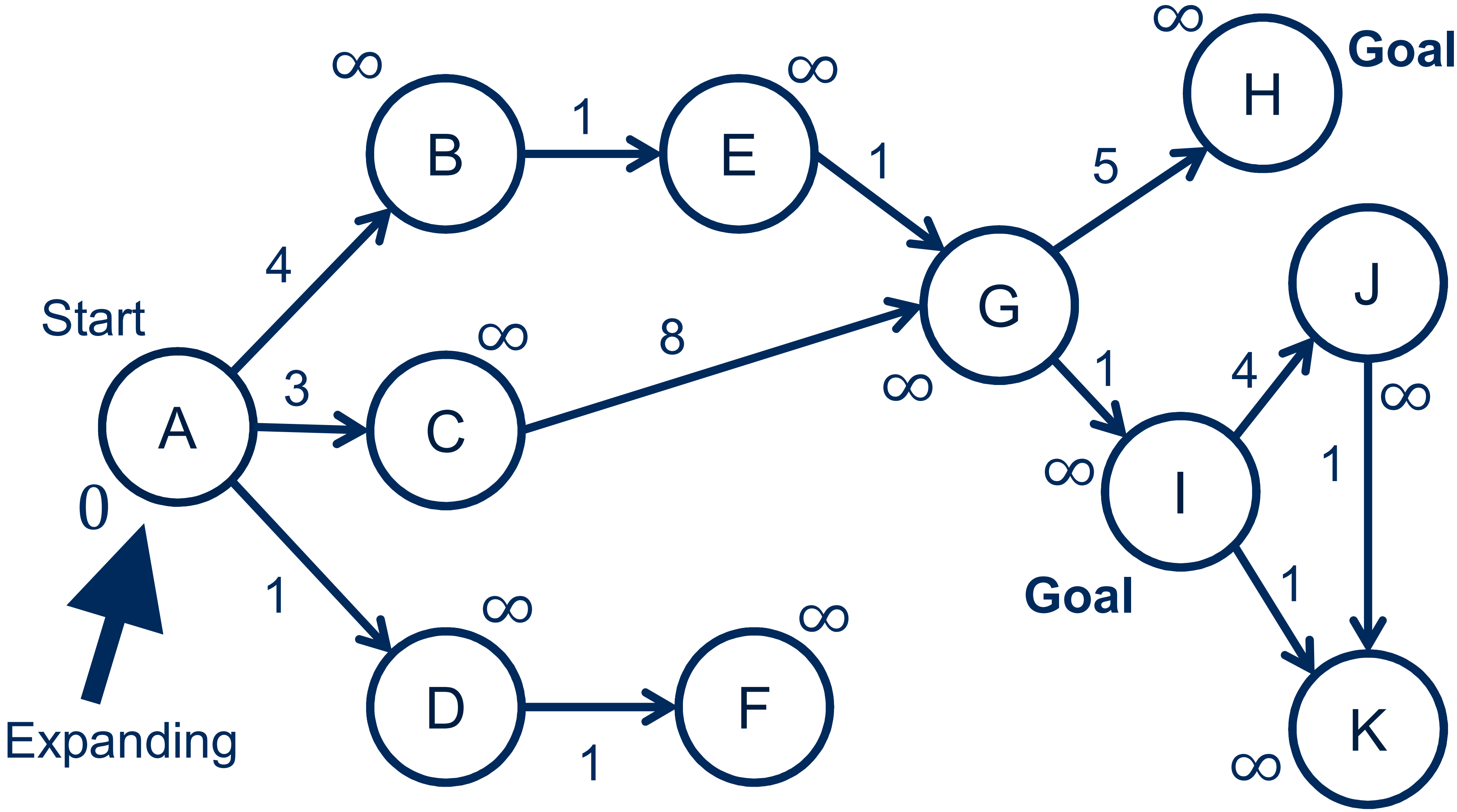

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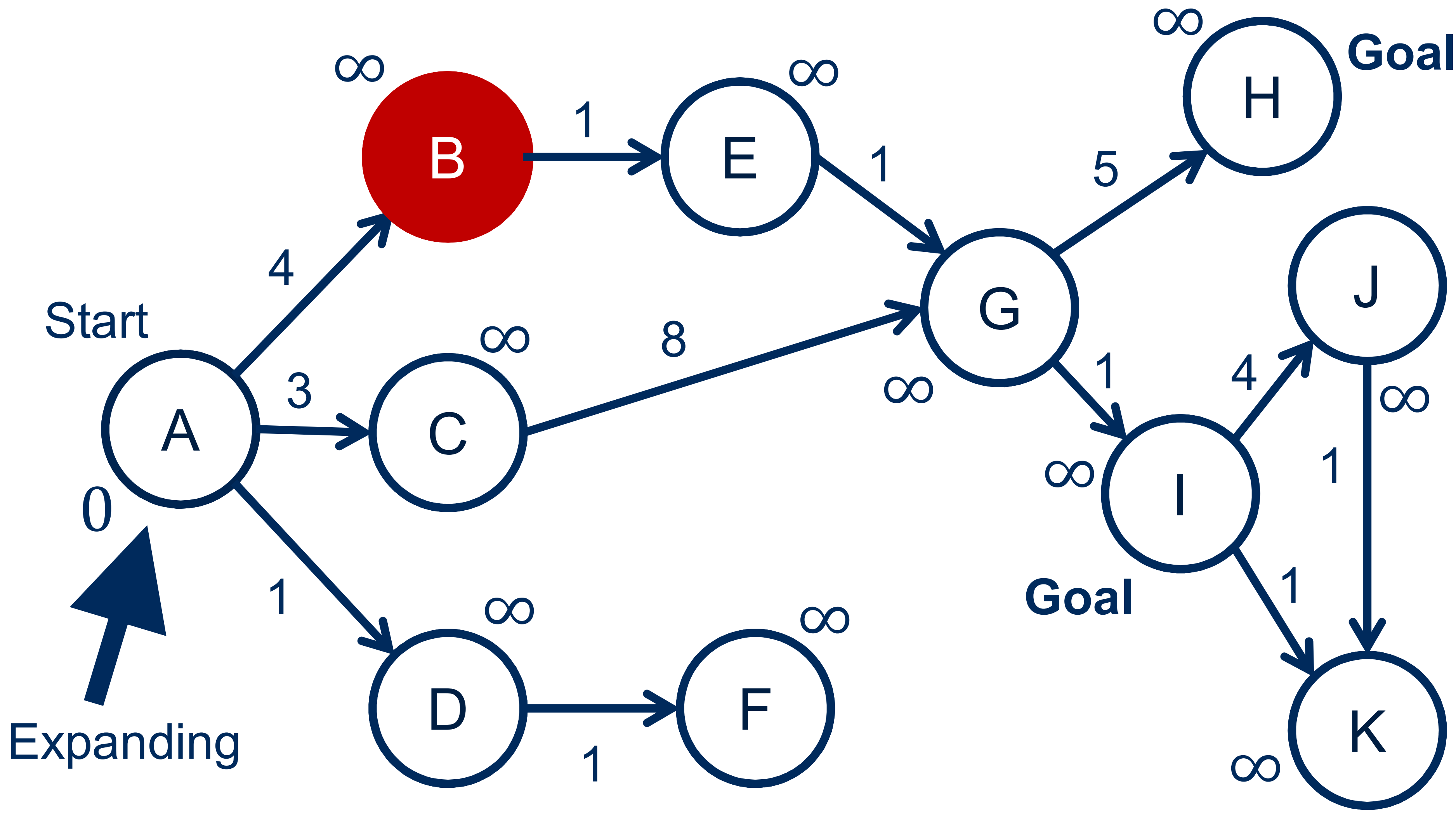

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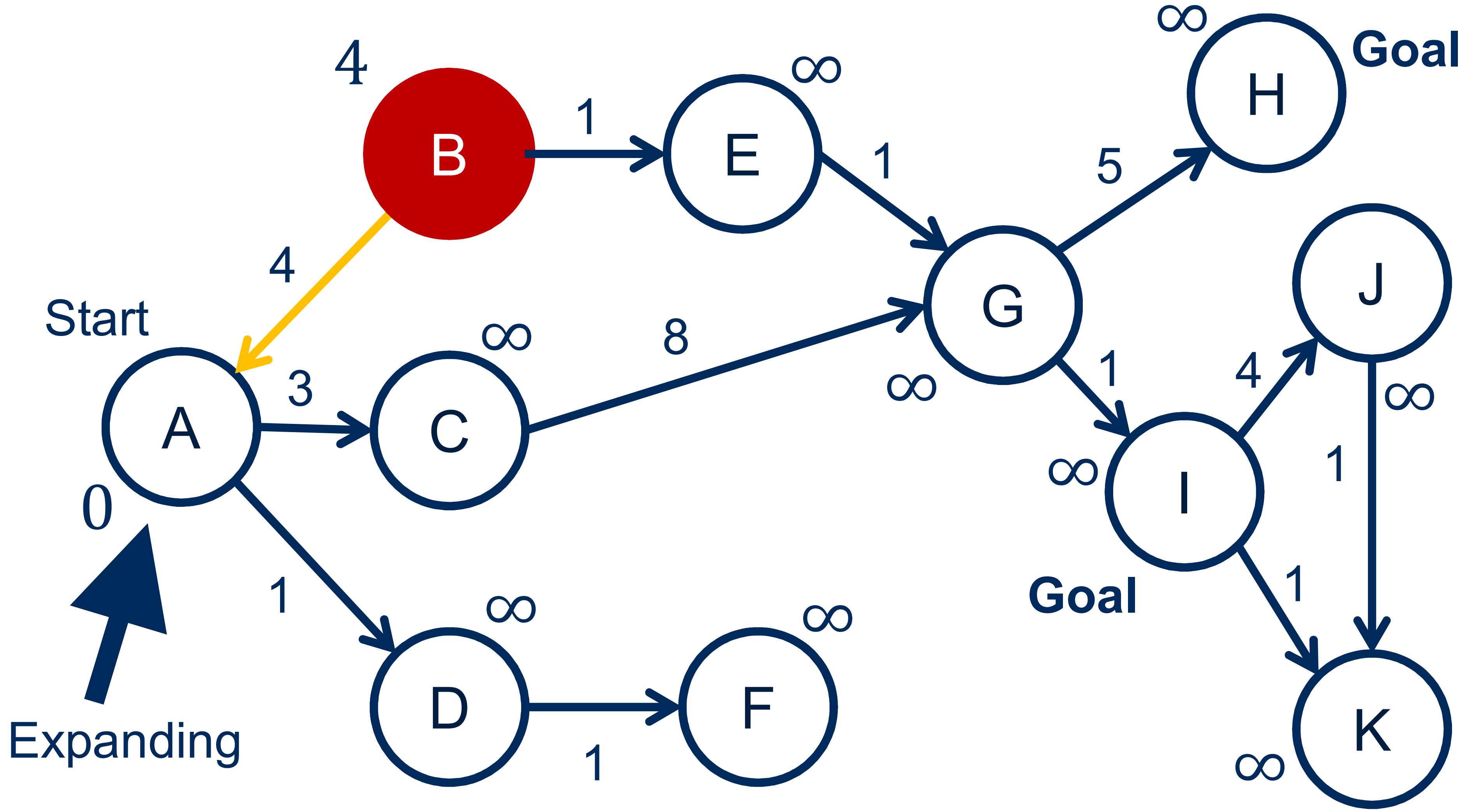

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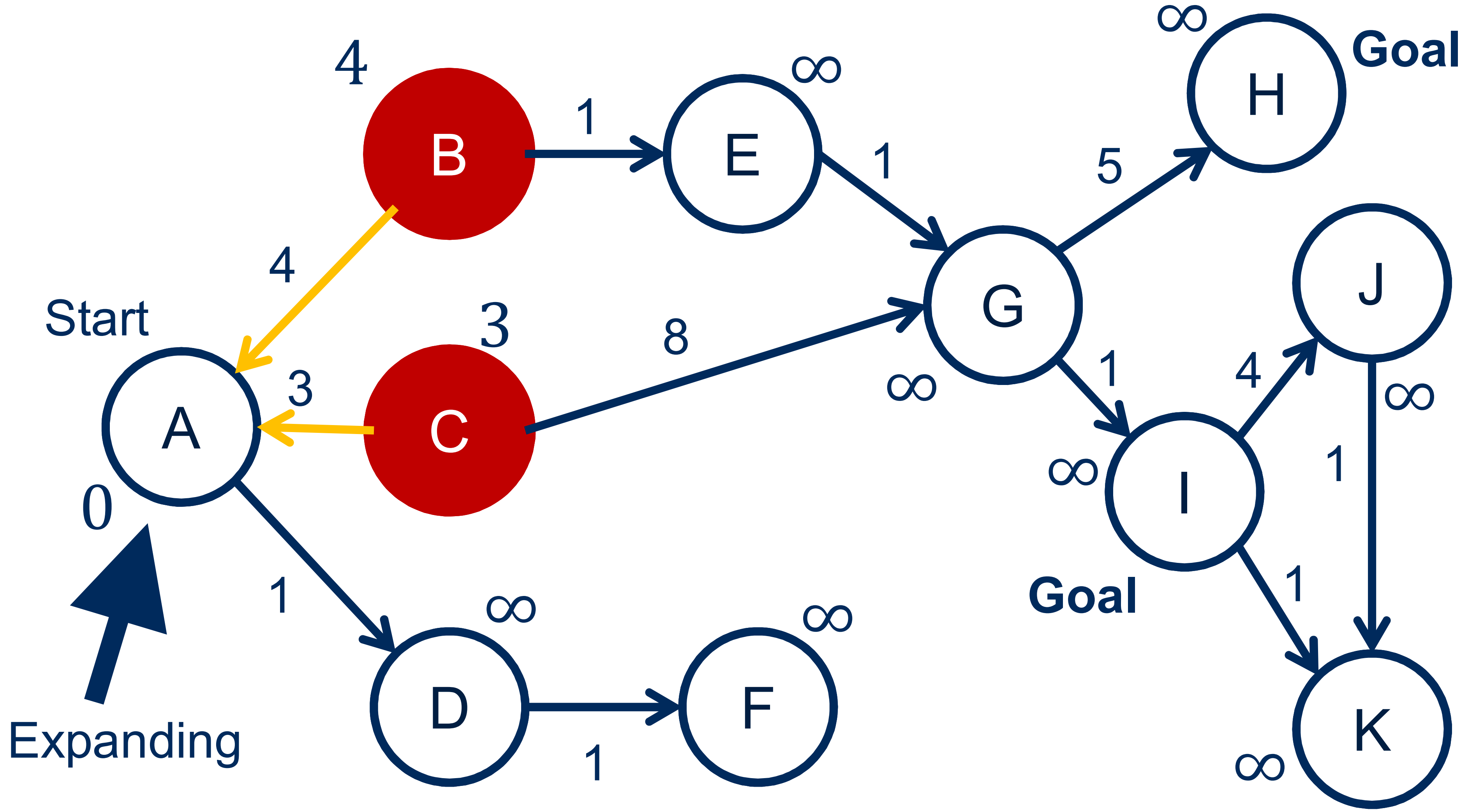

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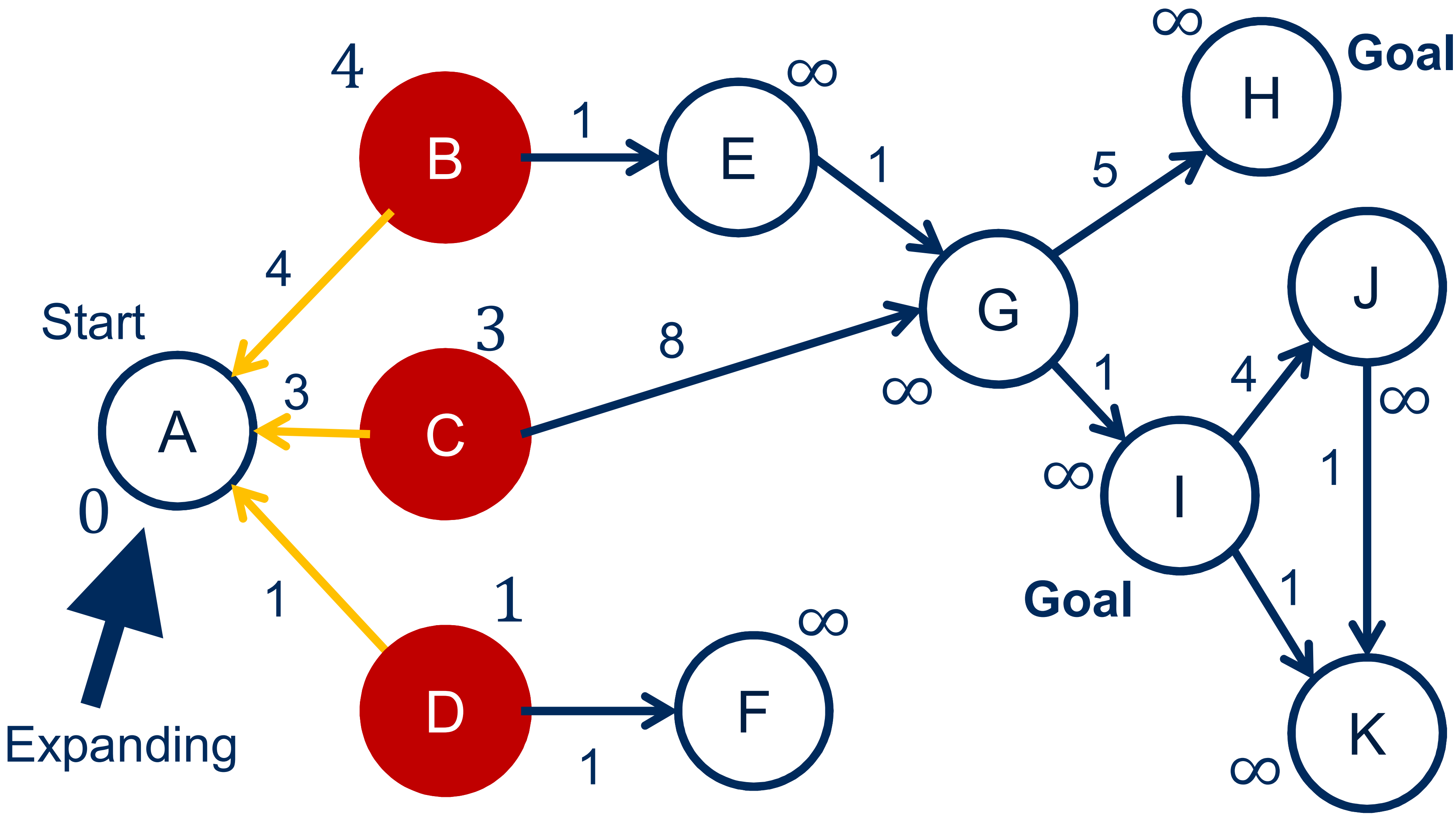

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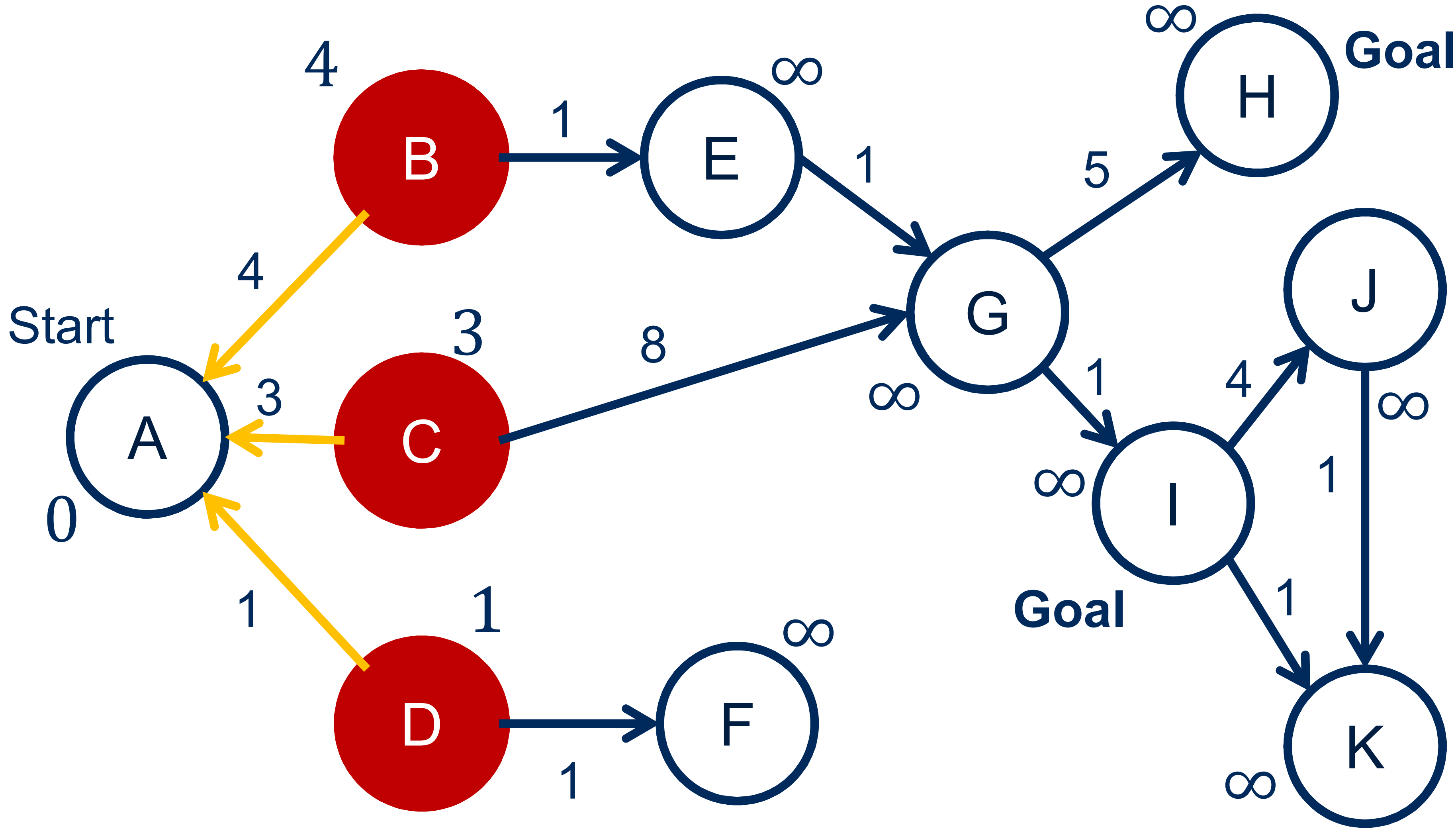

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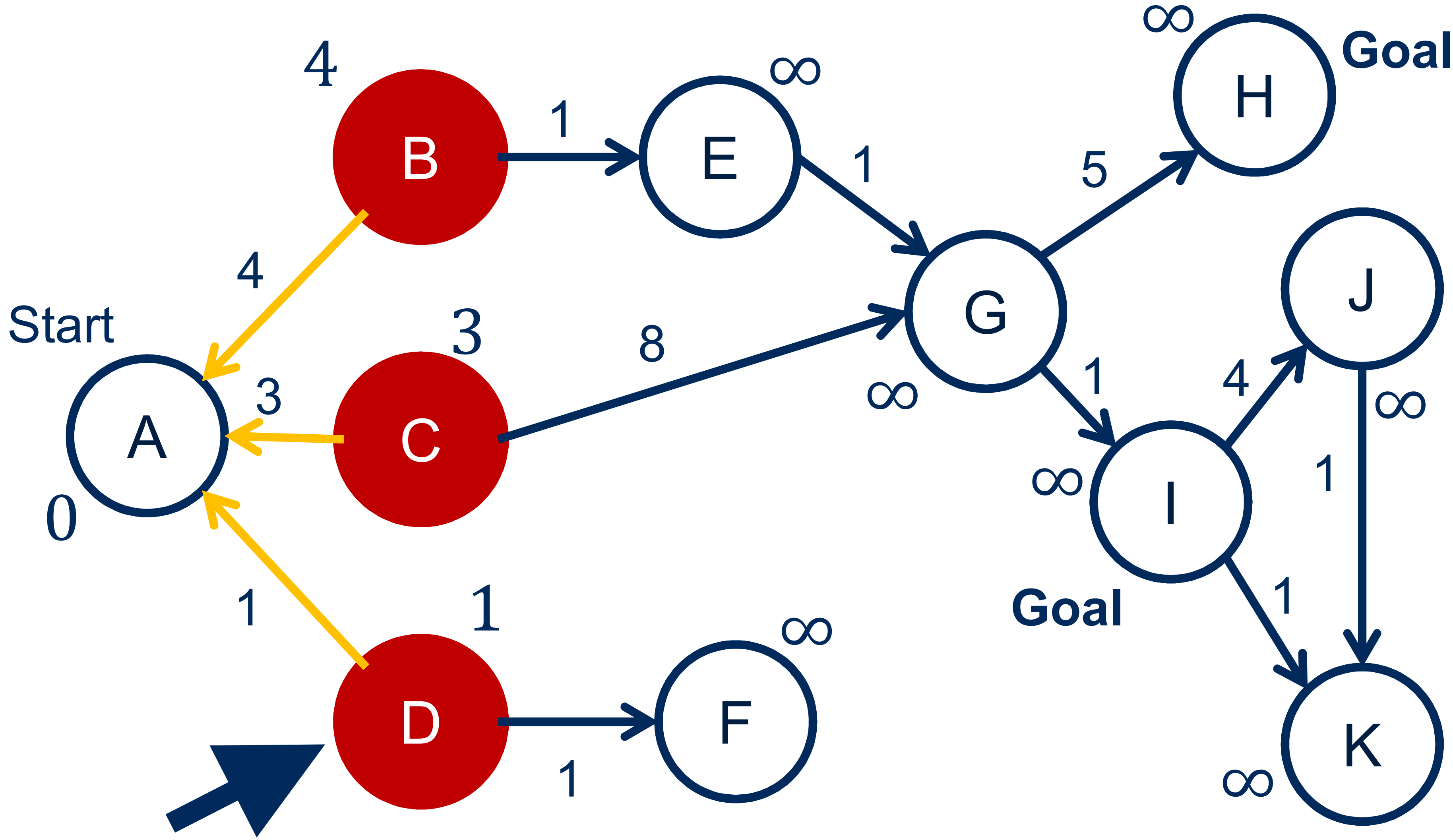

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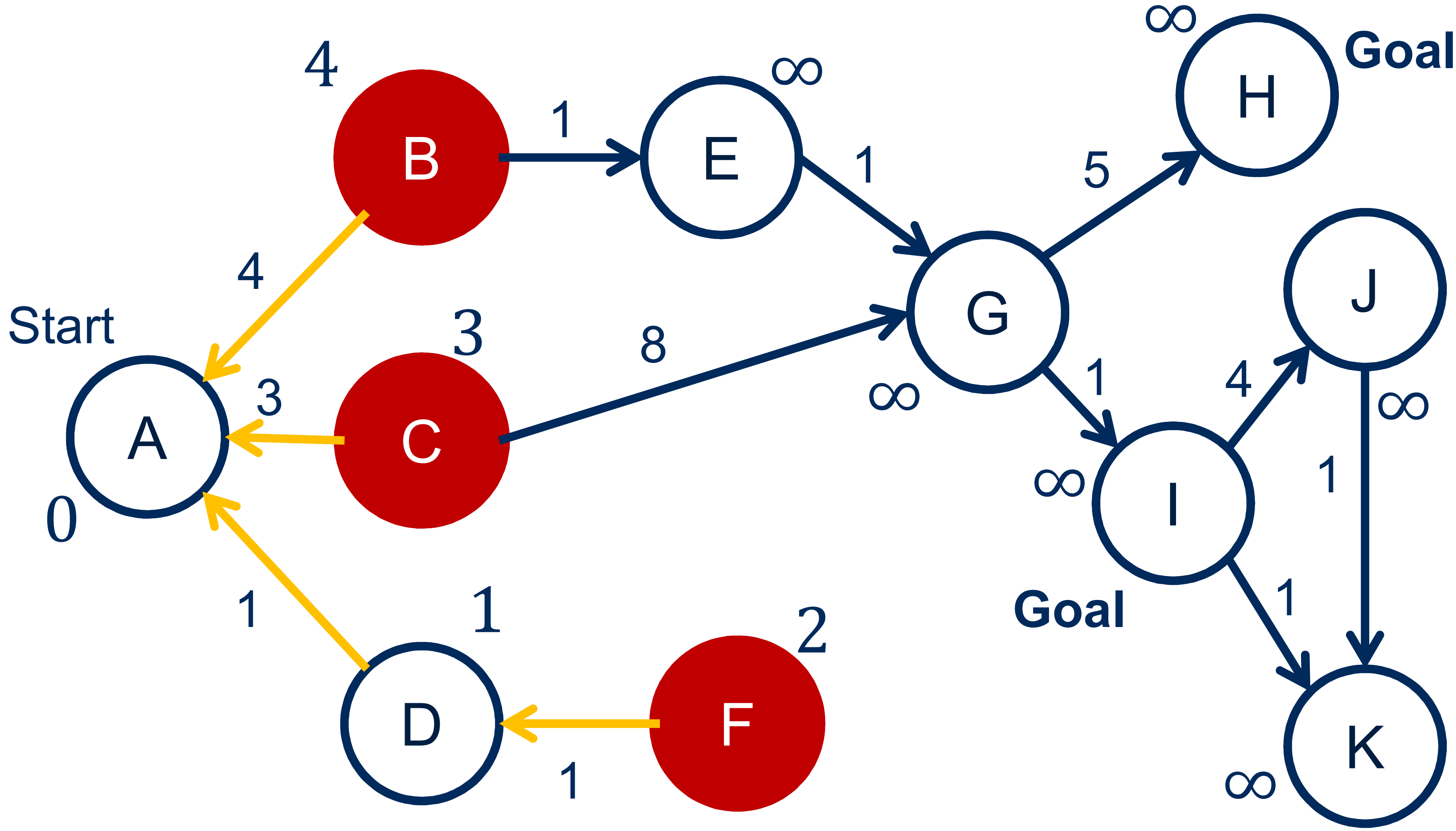

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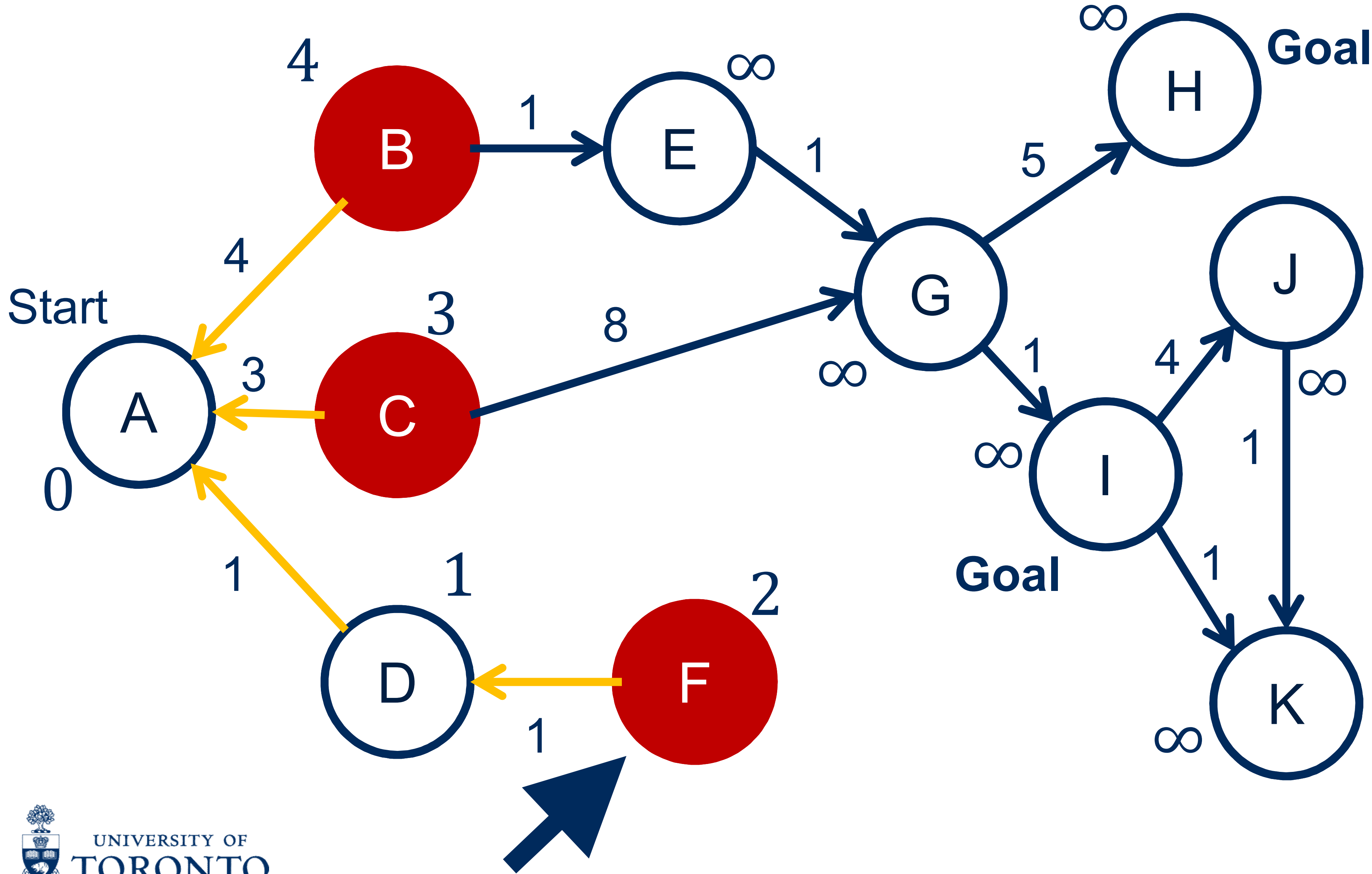

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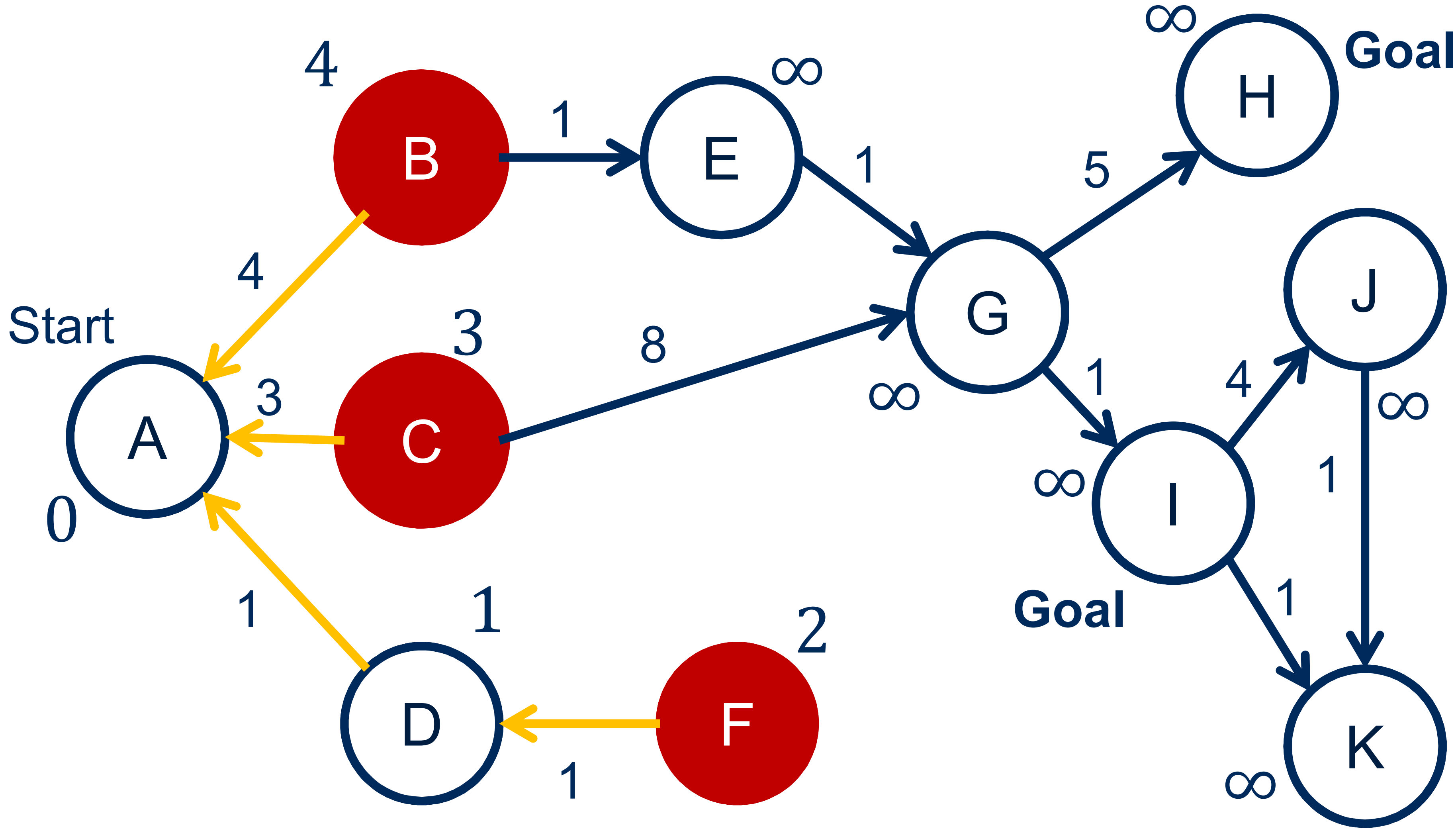

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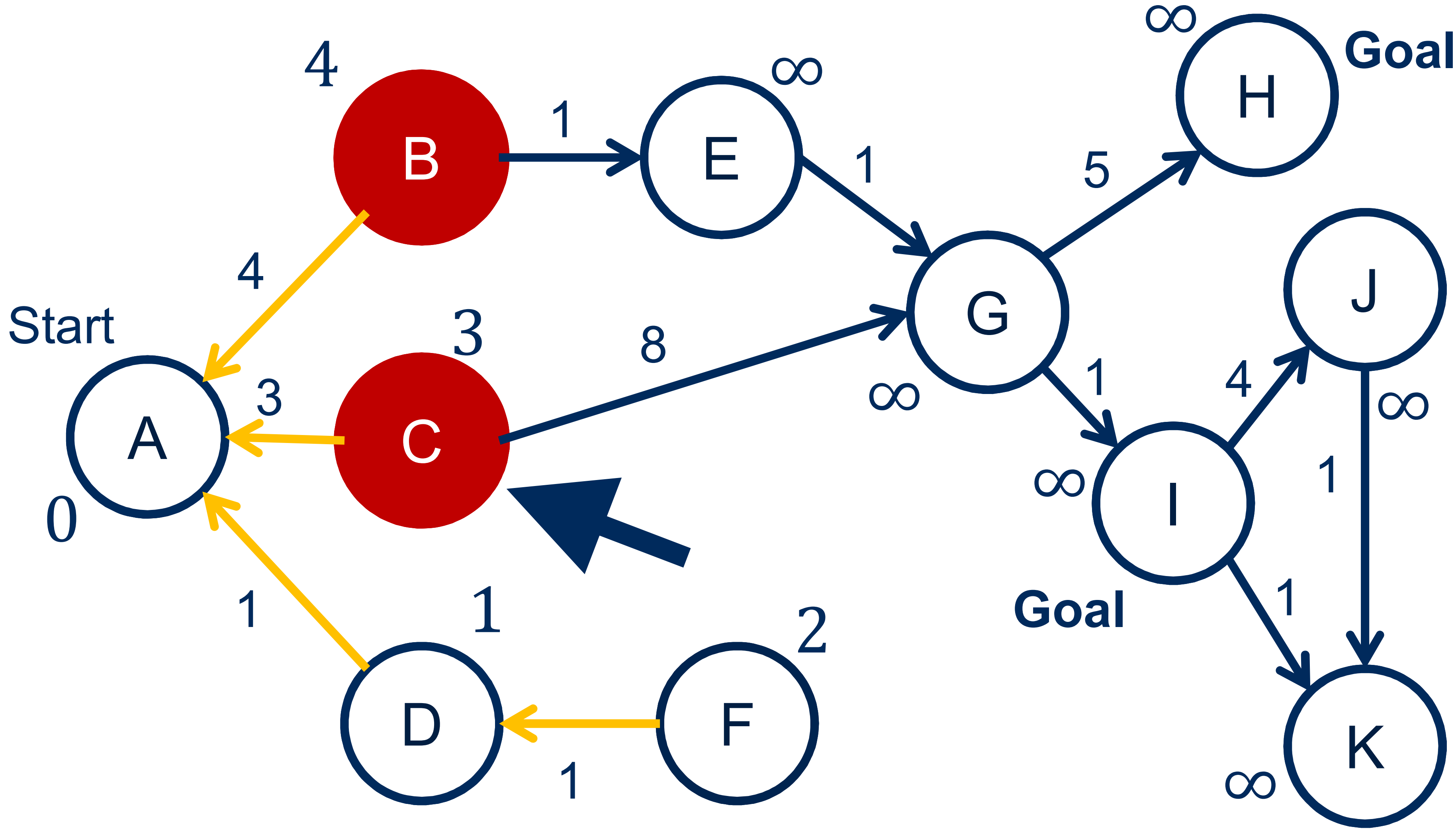

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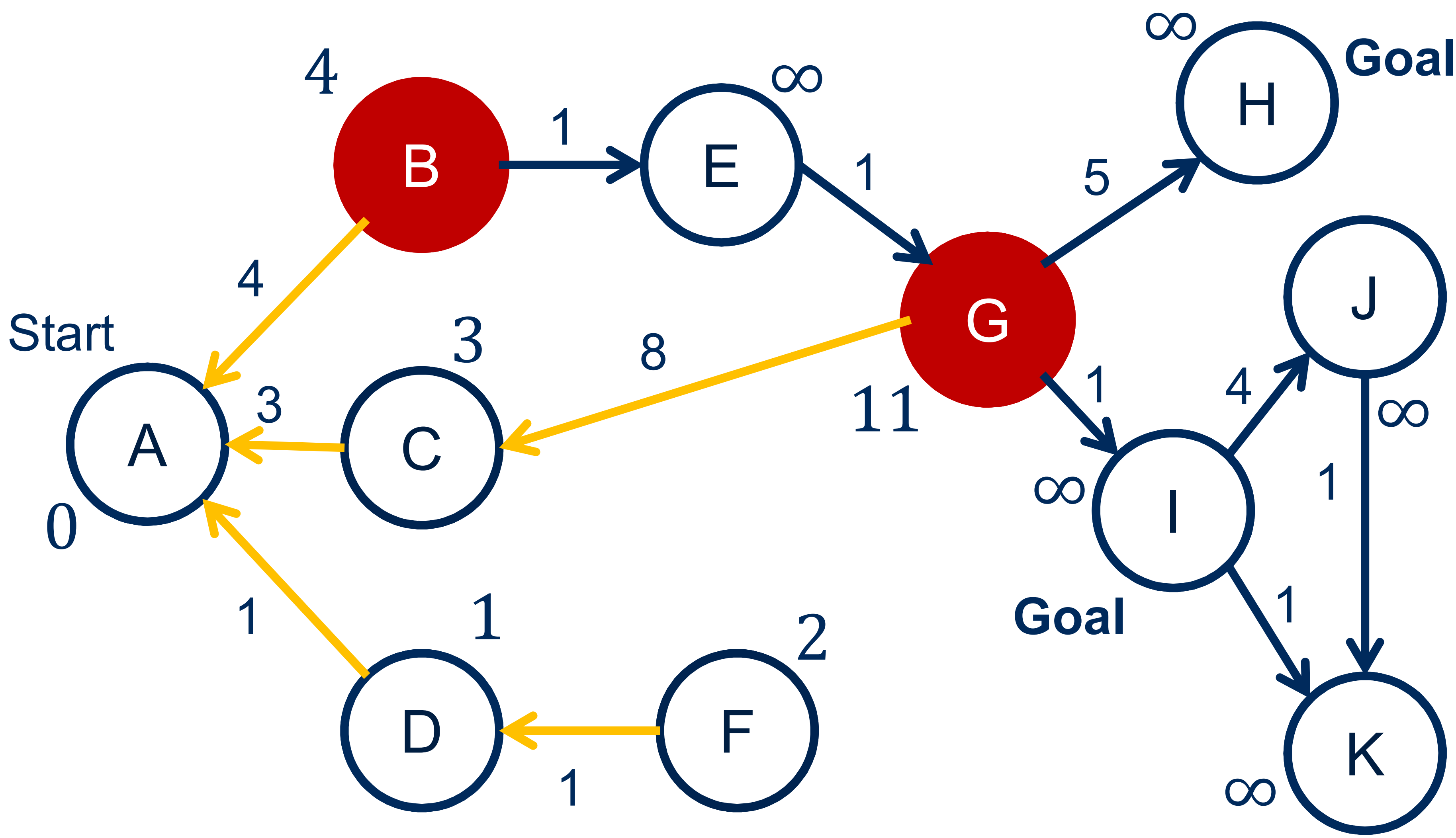

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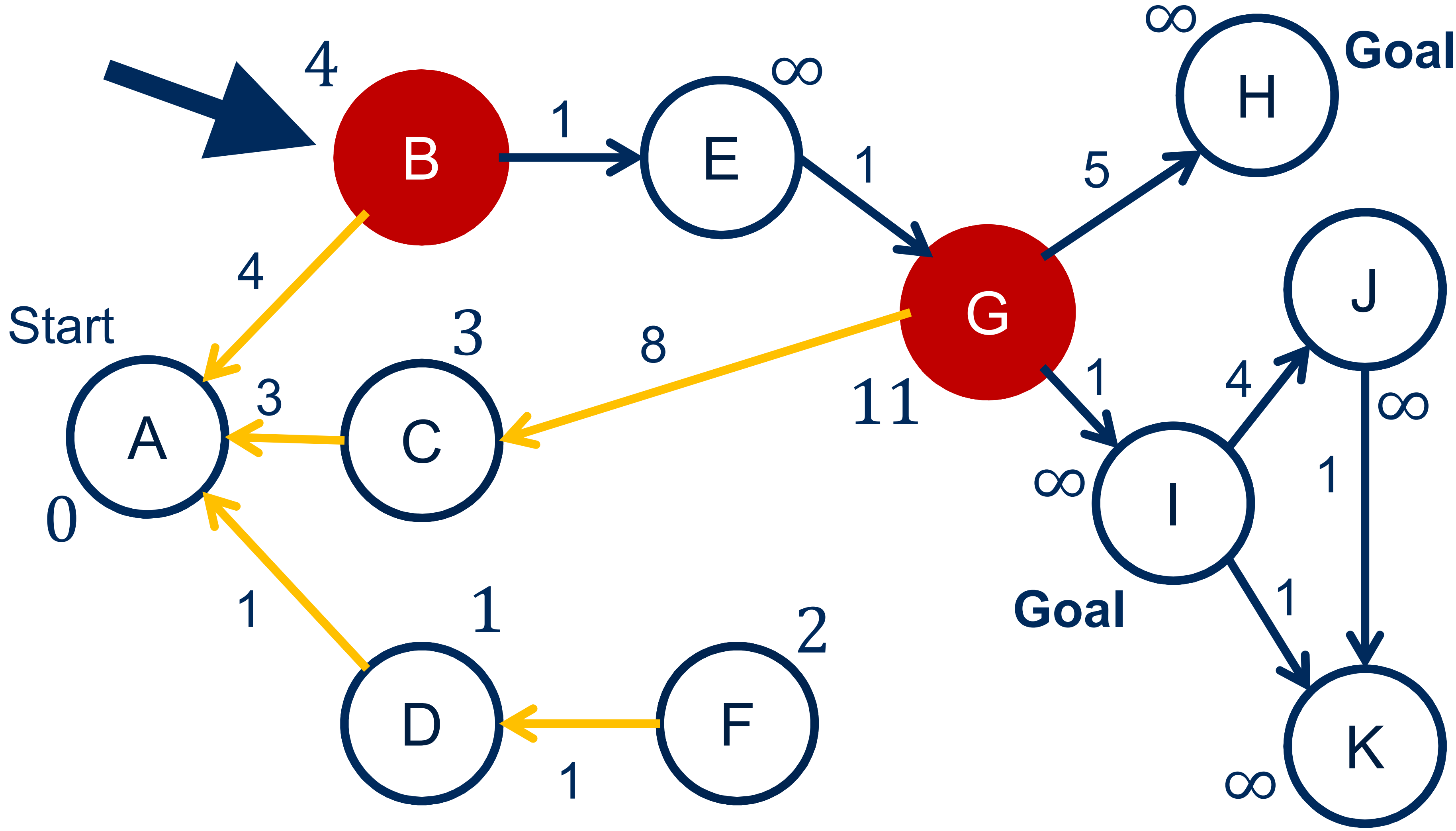

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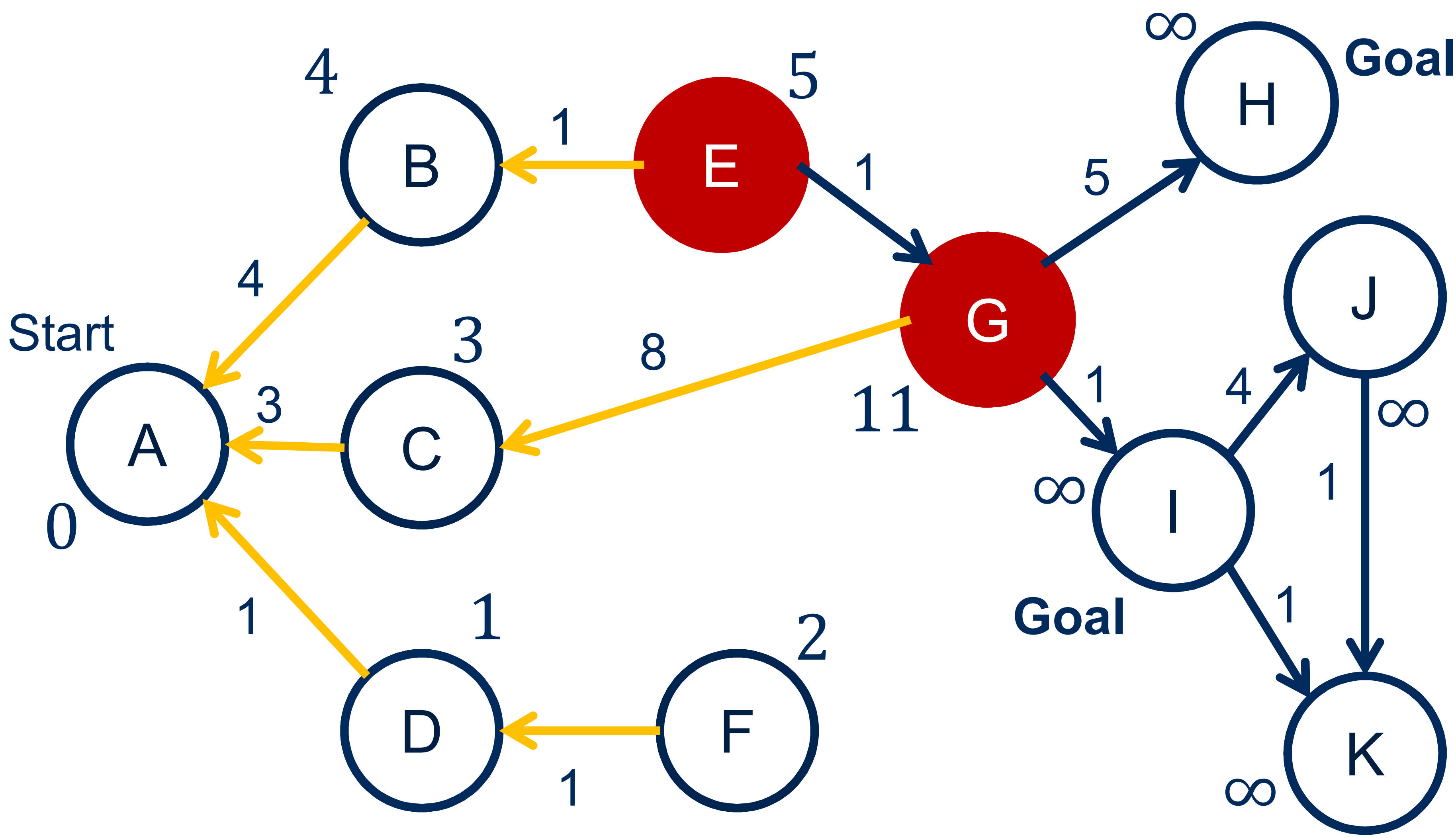

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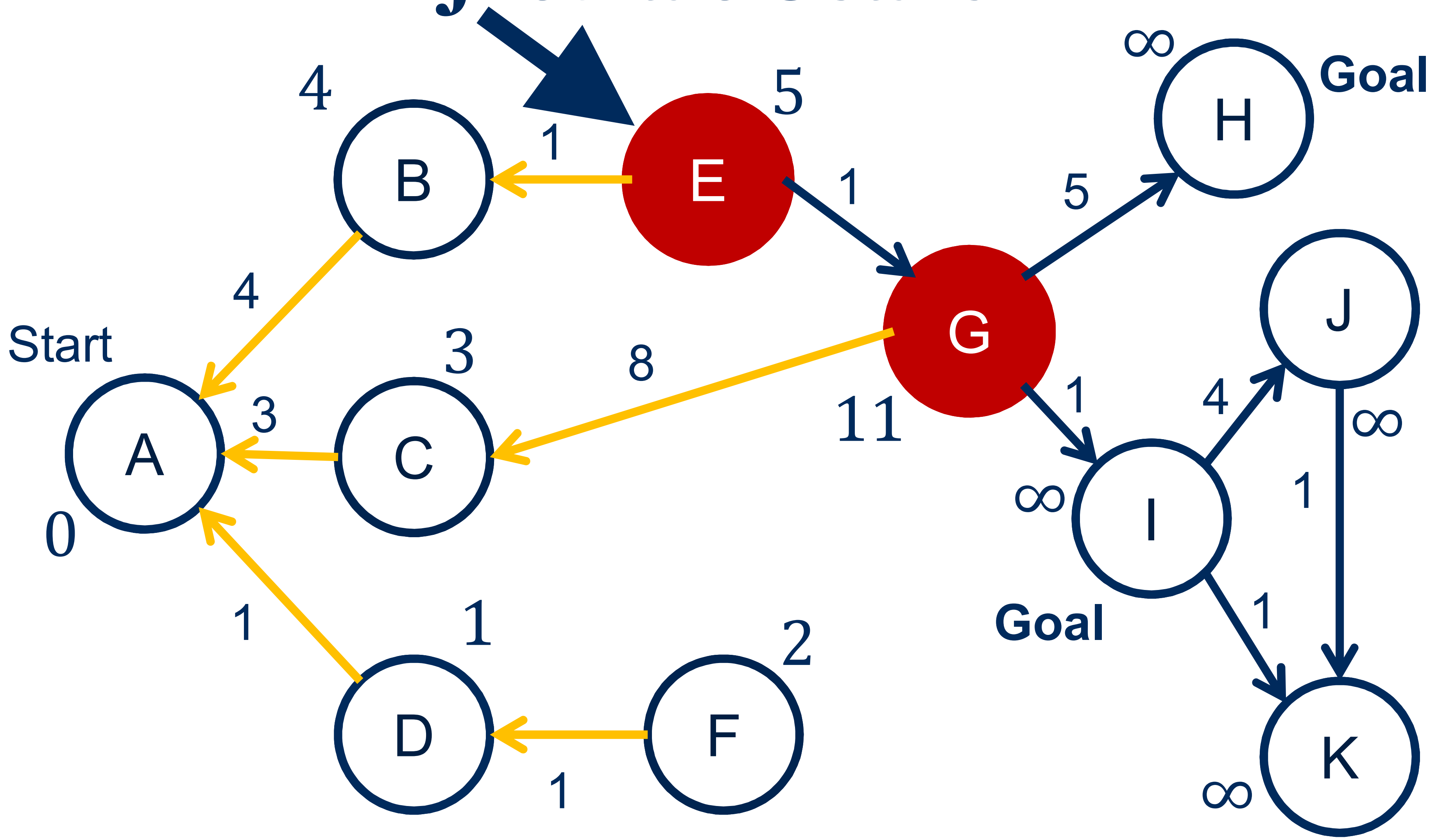

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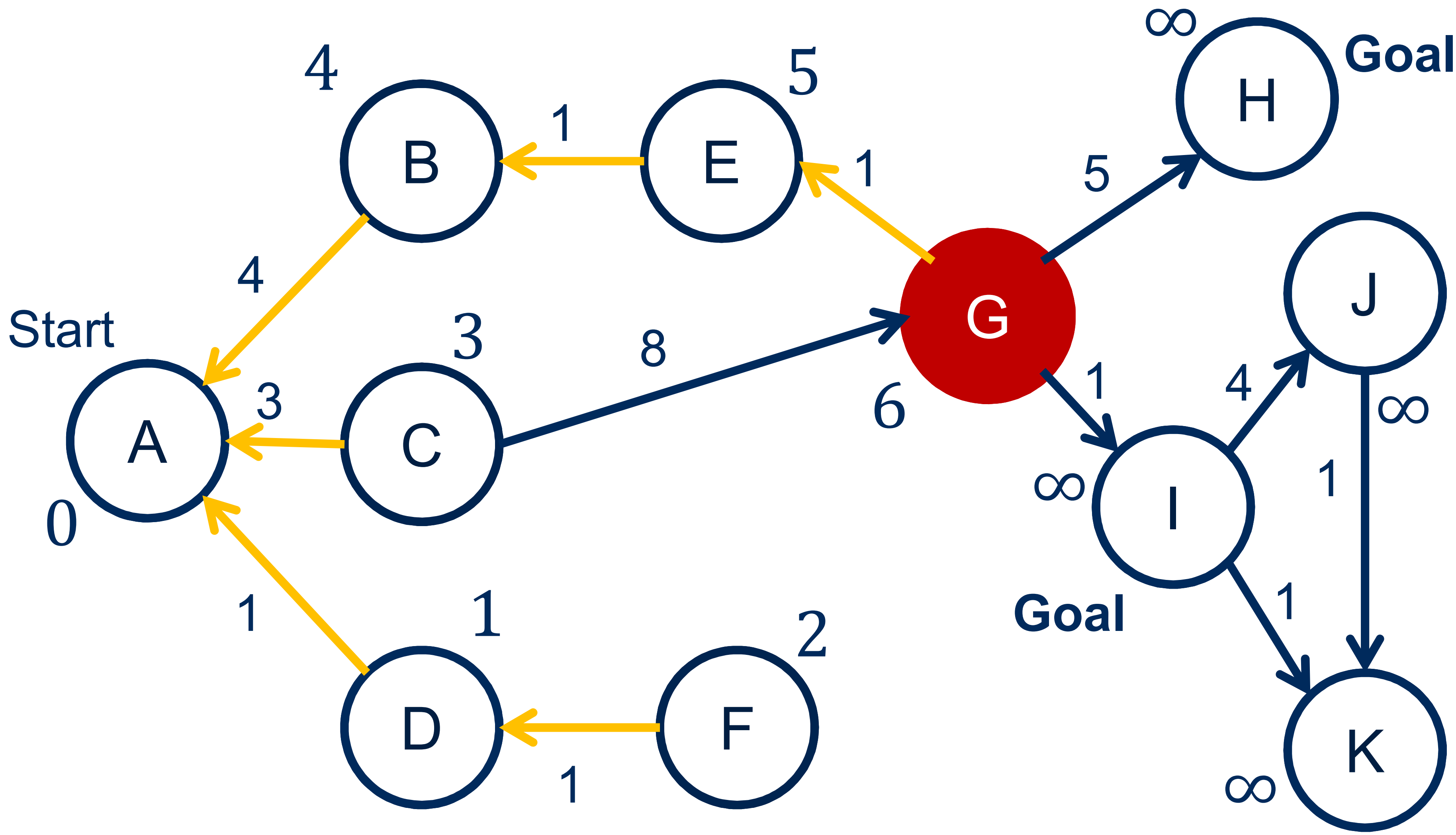

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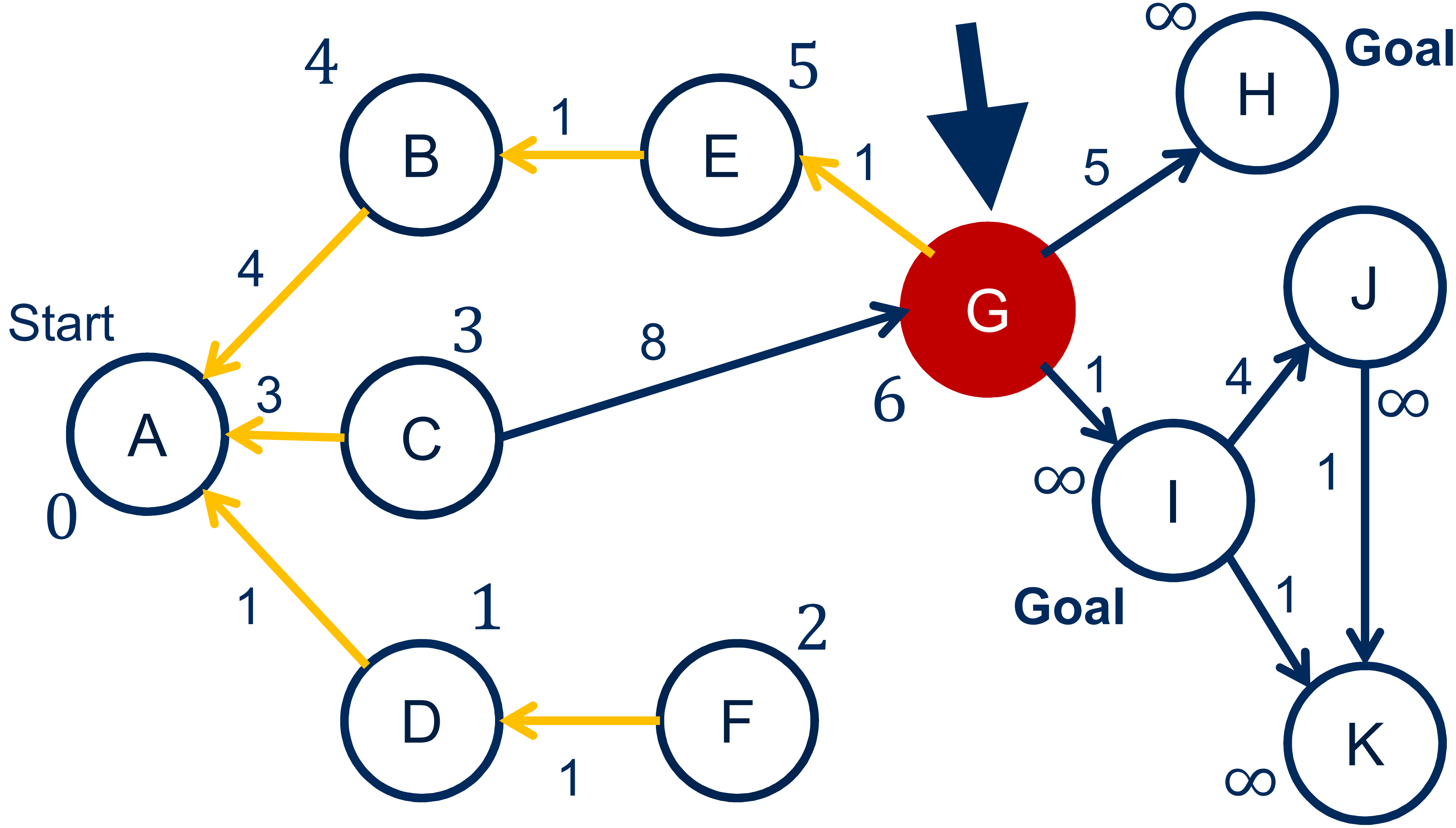
Dijkstra’s Search



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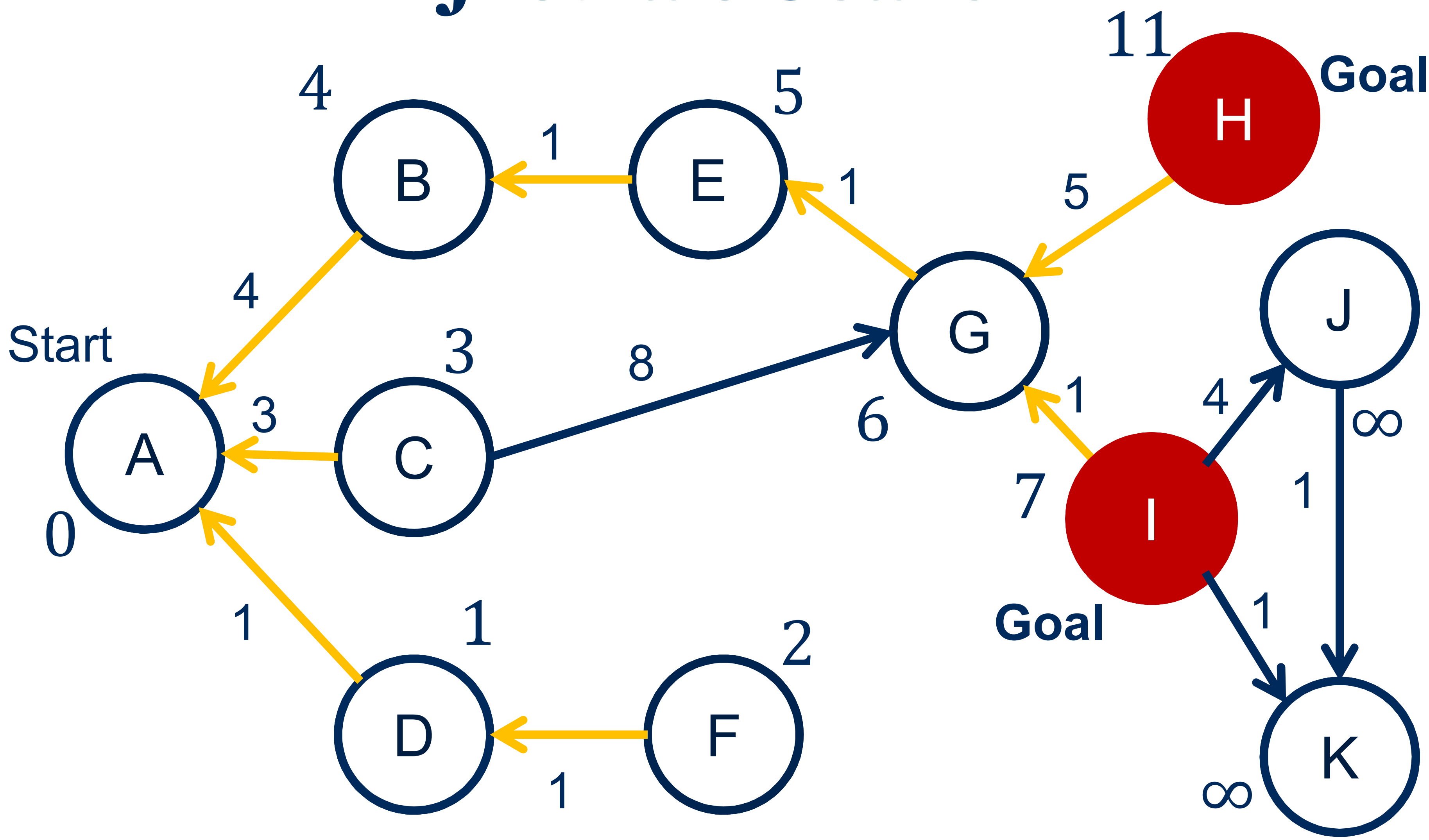

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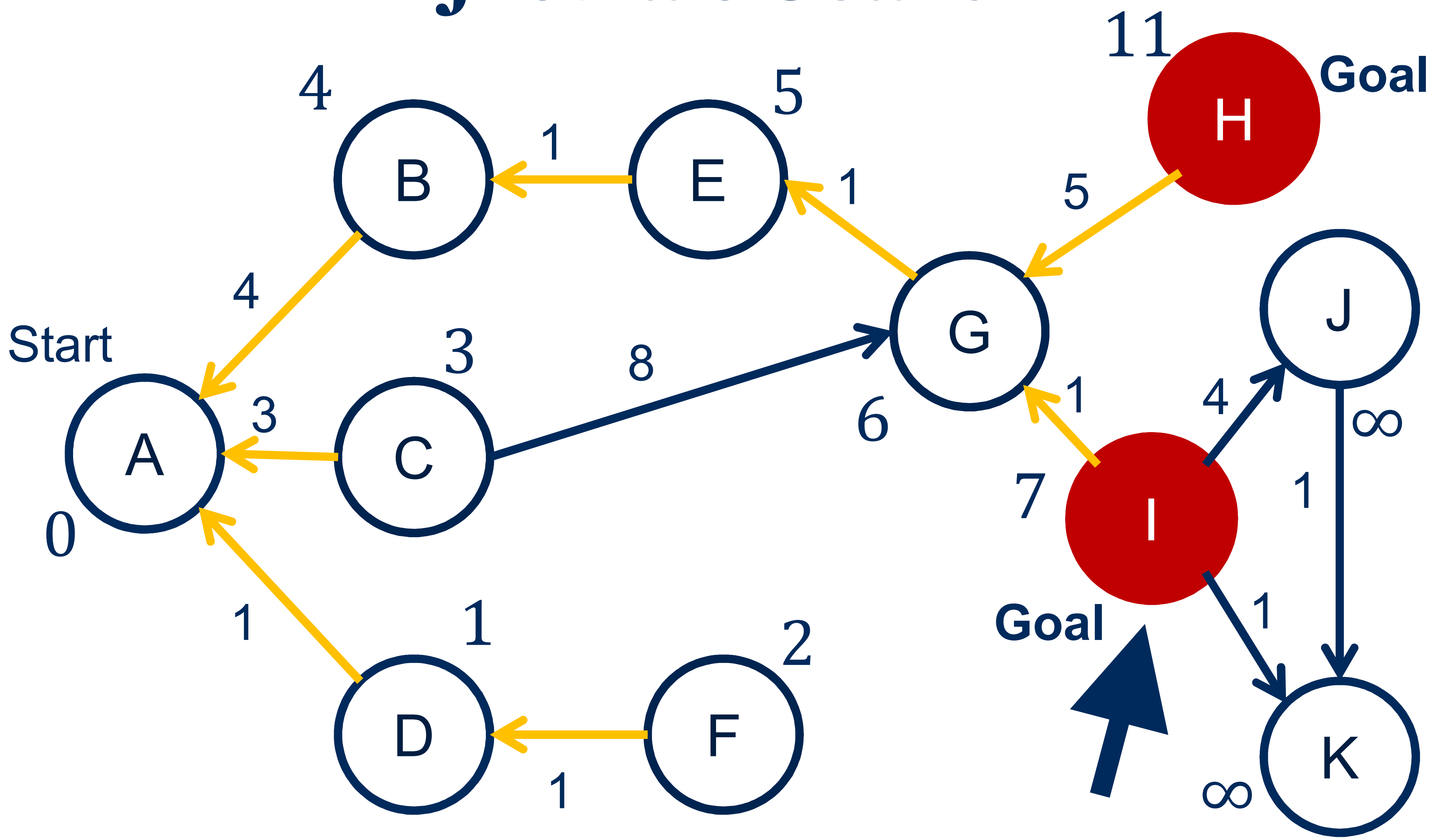

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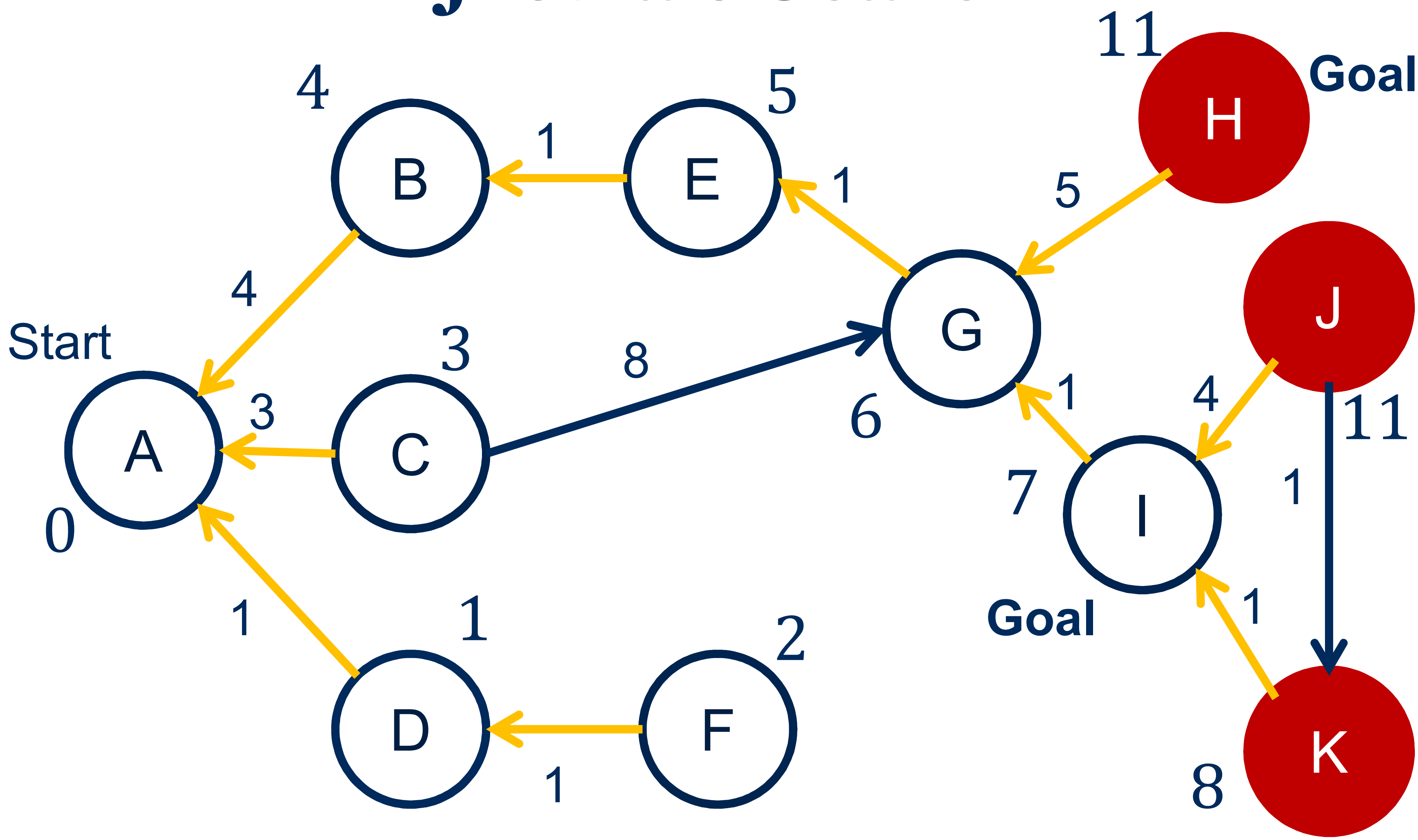

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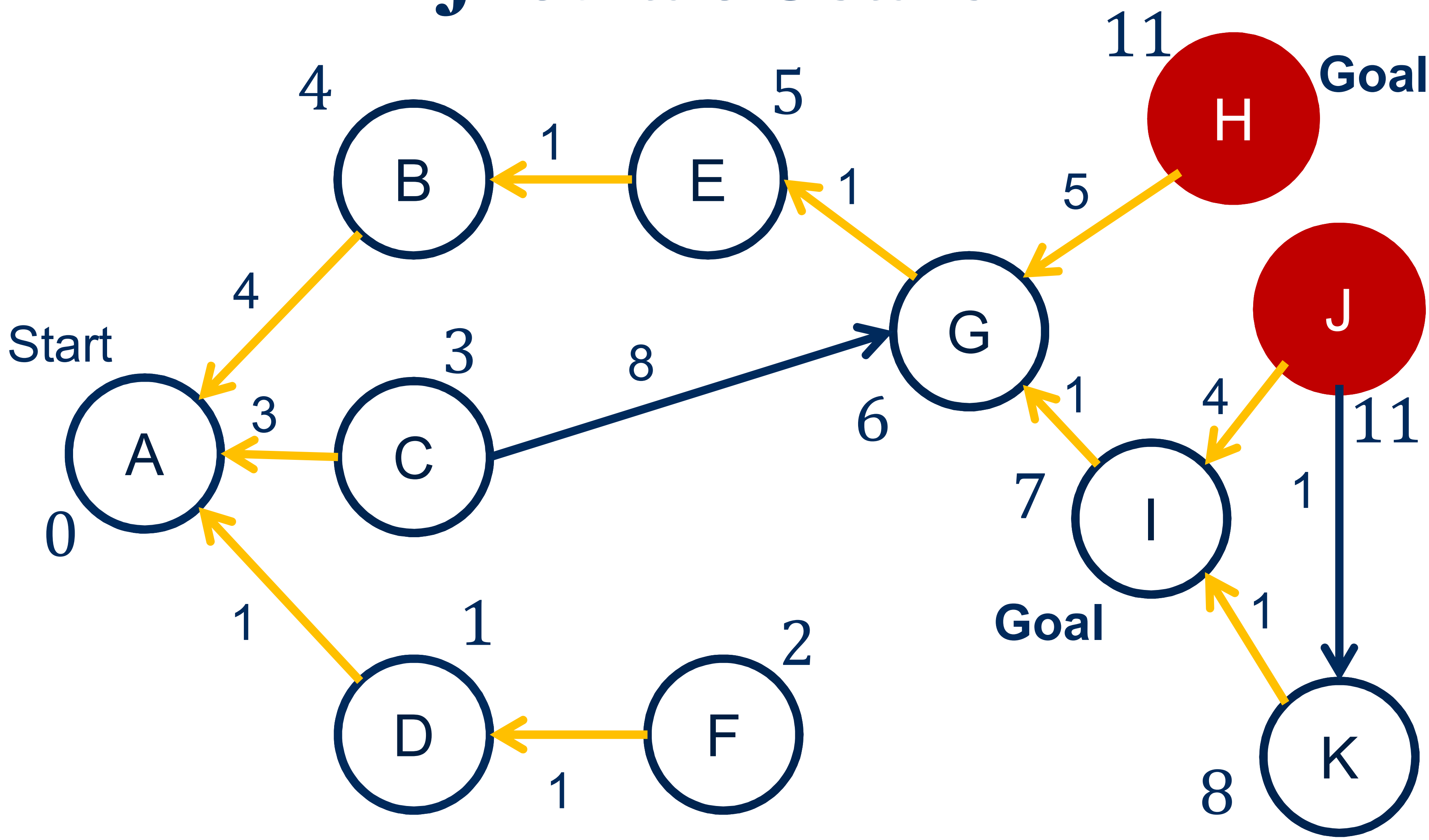

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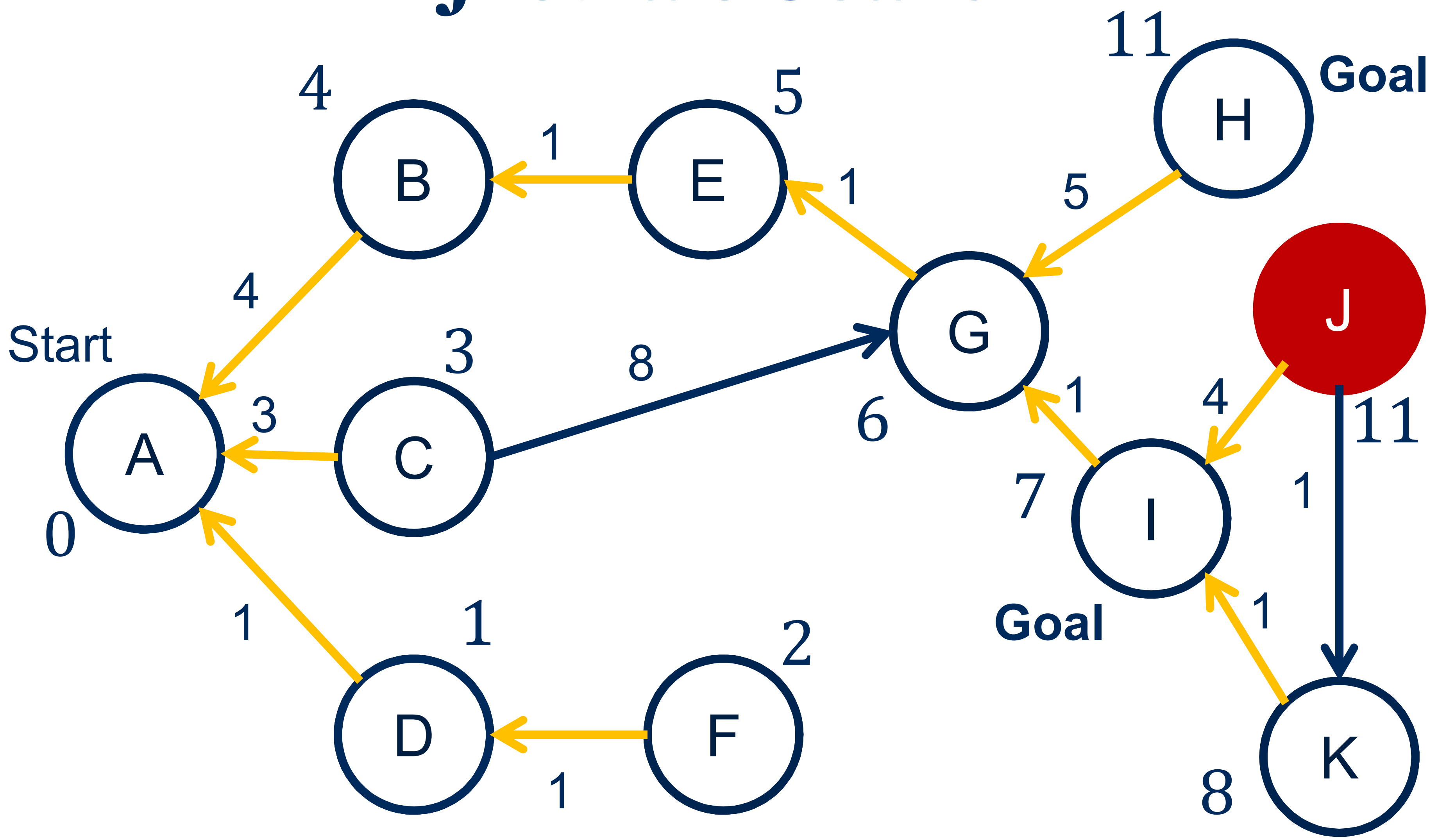

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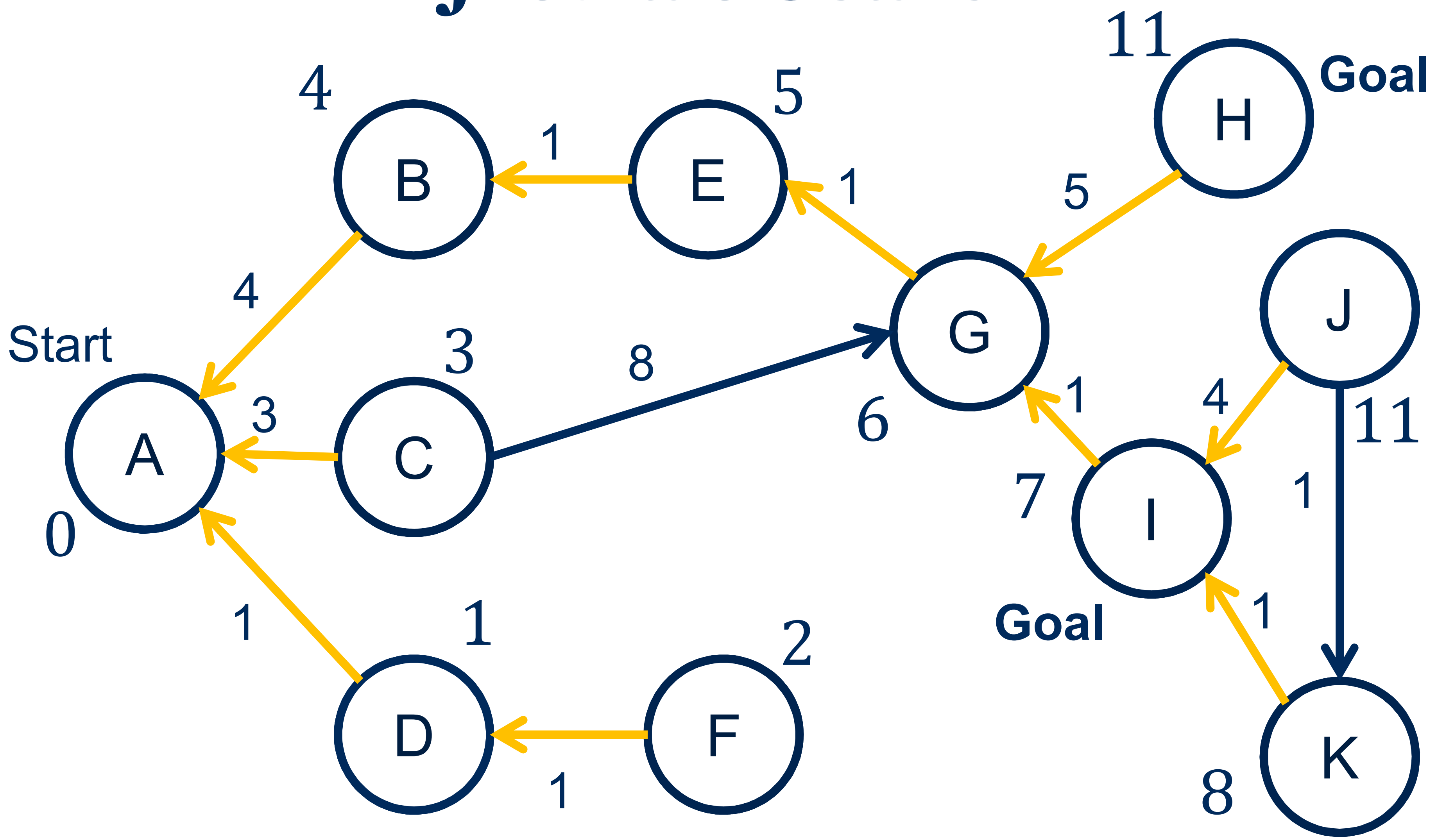

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Dijkstra's Search

- Now have a shortest path to every vertex in graph
  - Can iterate through goals and return lowest-cost solution

- Dijkstra's search will look at O(|V|) vertices
  - So planning can be done in poly-time, right?

Dijkstra's Search

- Dijkstra's search is polynomial in |V|
  - But not polynomial in the given problem representation

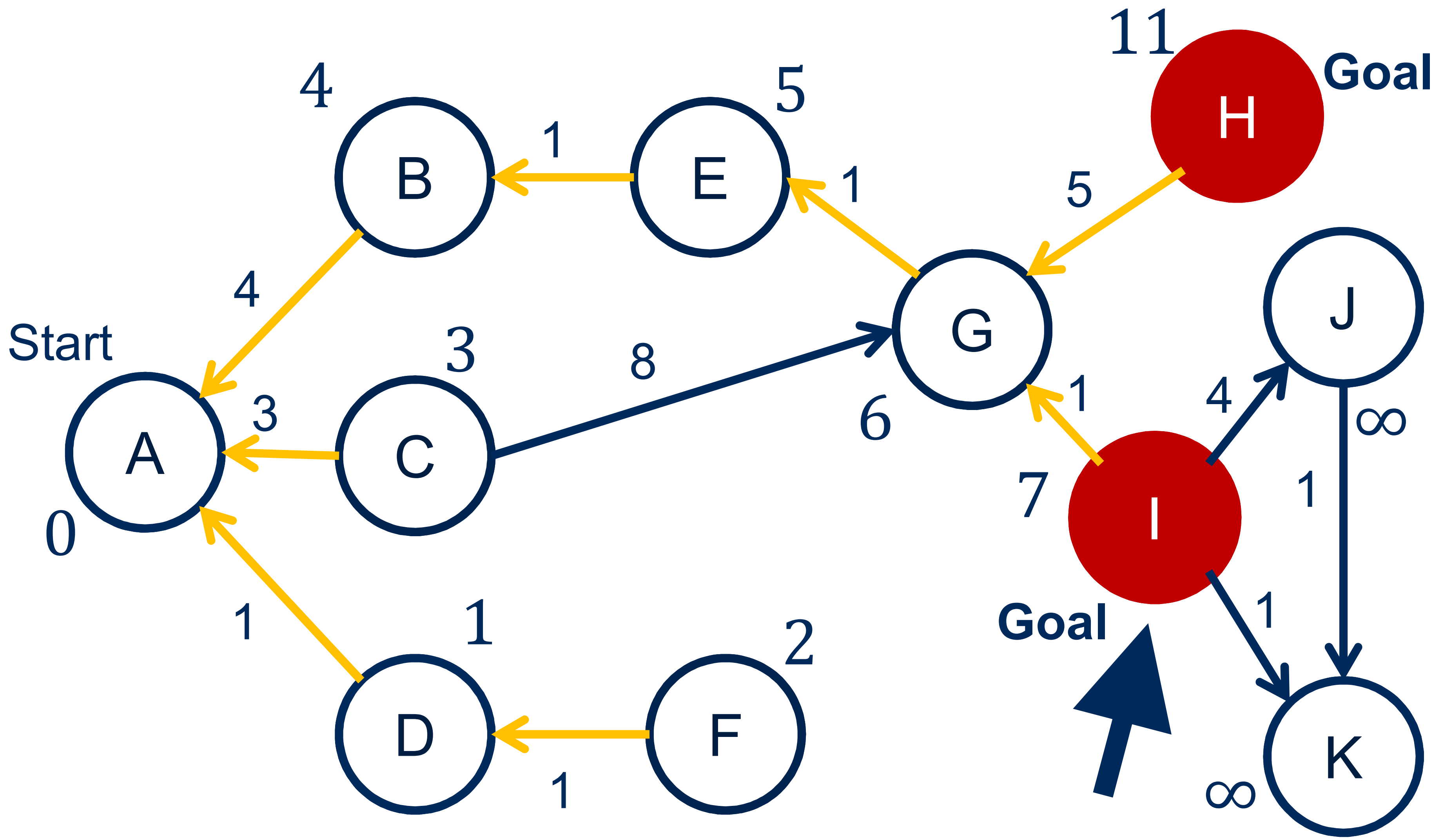
- Consider floortile on an N x N grid with K locations that need to be painted
  - Robot can be in either LOADED-B or LOADED-R
  - Robot can be in any of N x N locations
  - Any combination of the K locations can be painted
  - $O(2 \cdot N \cdot N \cdot 2^K)$ states

Uniform Cost Search

- Two changes to Dijkstra's Algorithm

1. Stop after a goal node is first expanded.

Uniform Cost Search



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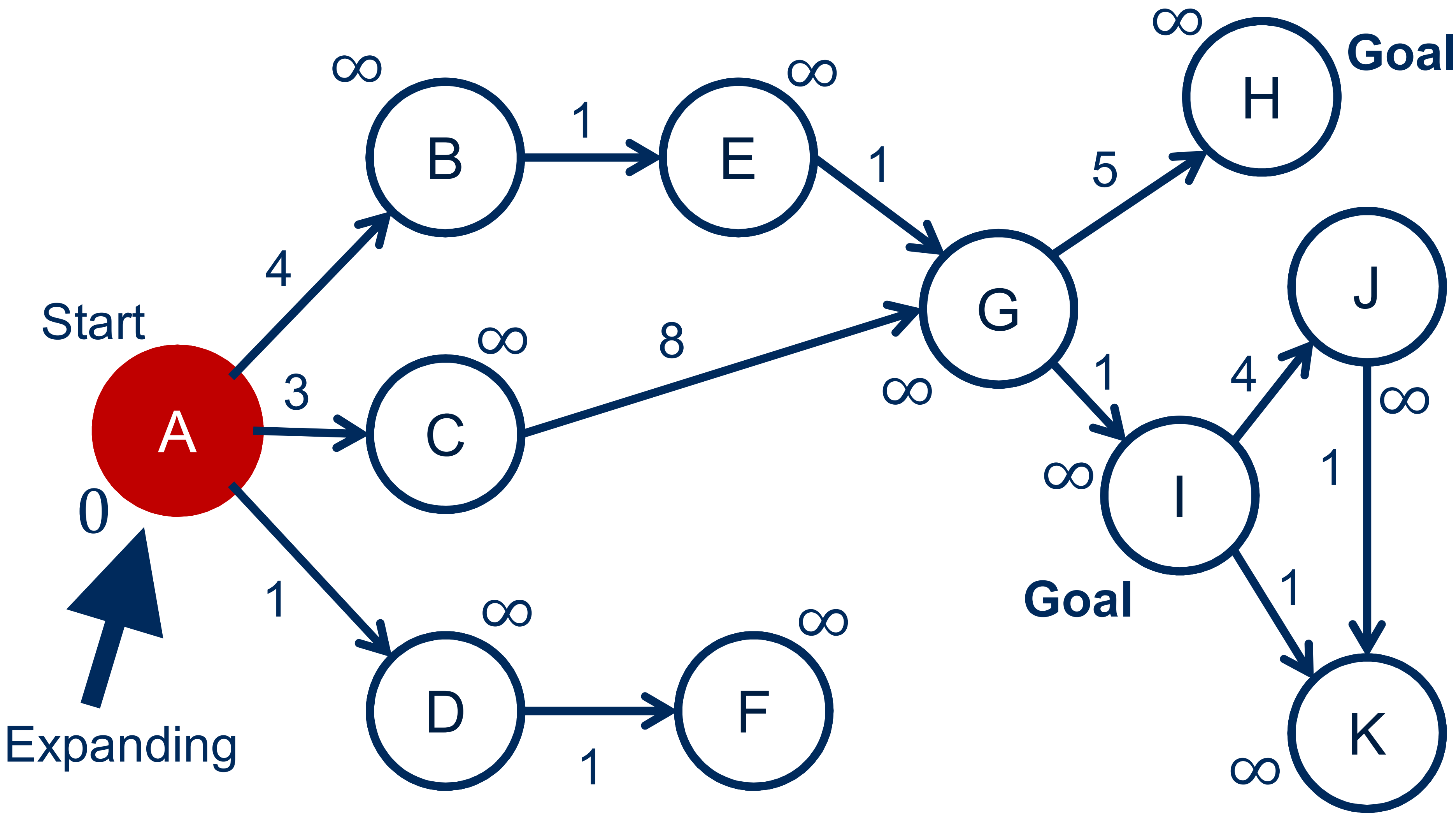
Uniform Cost Search

- Two changes to Dijkstra's Algorithm

1. Stop after a goal node is first expanded.

2. Use implicit action definition to generate the graph on-the-fly.

Dijkstra's Search



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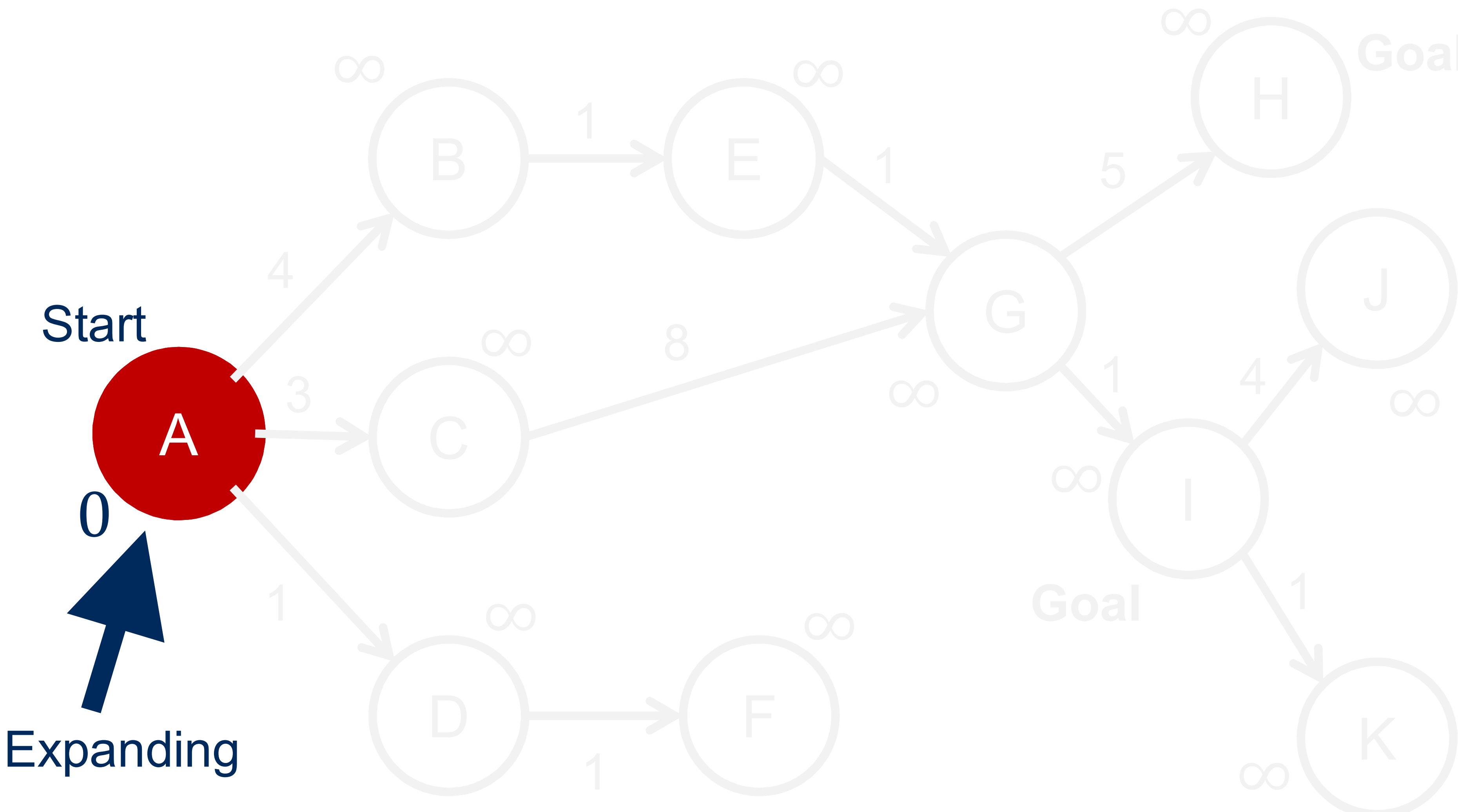

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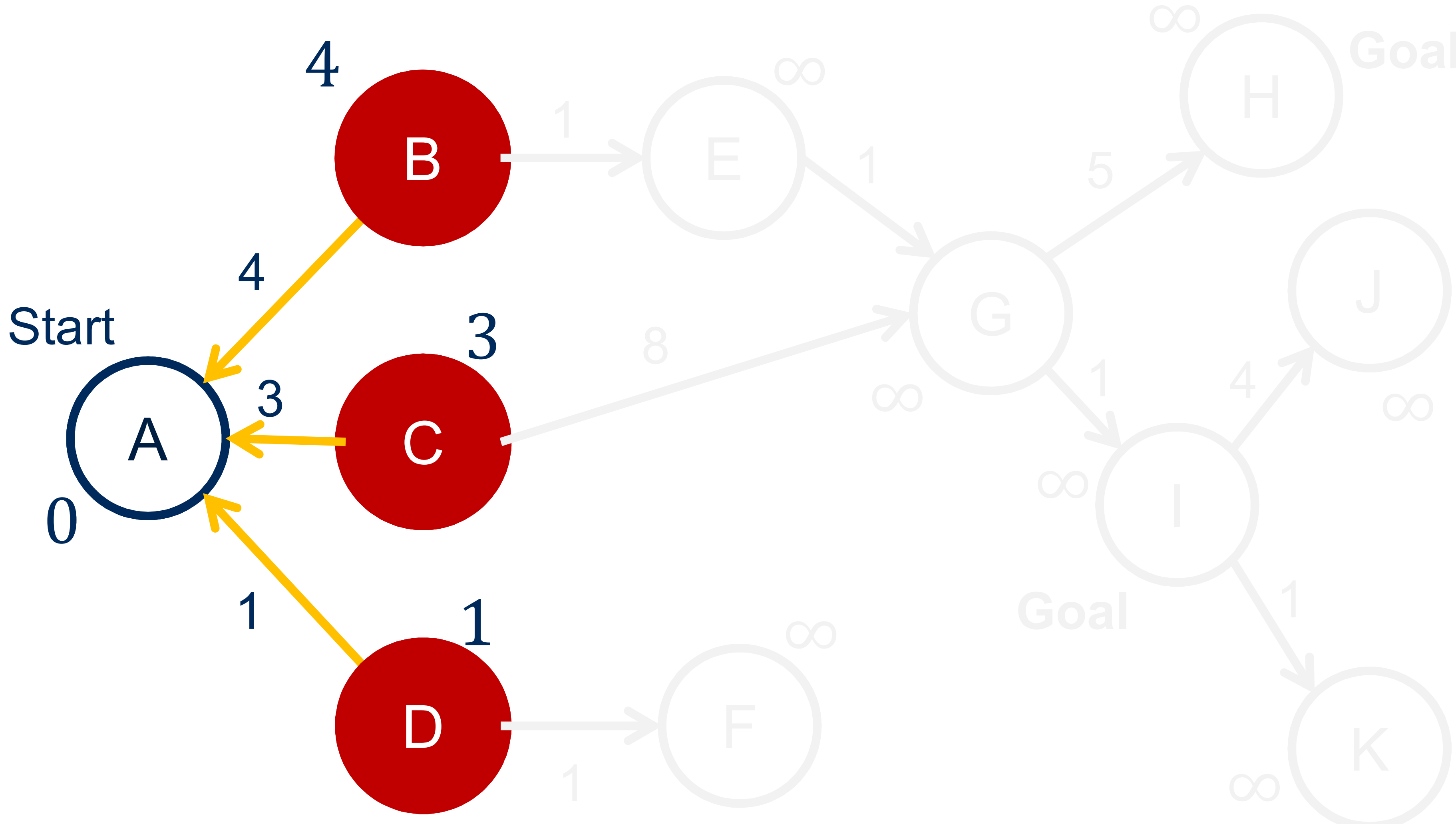

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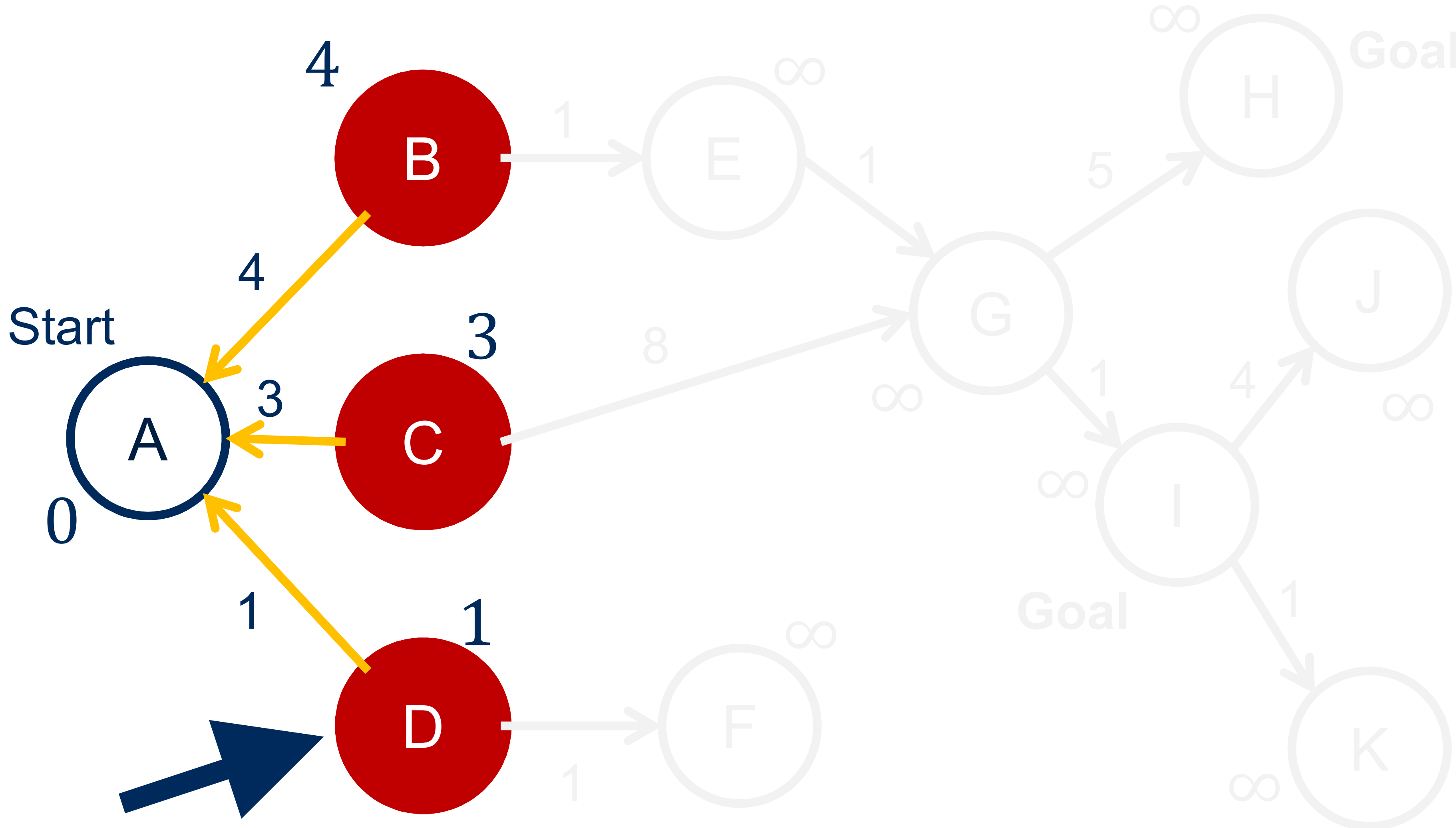

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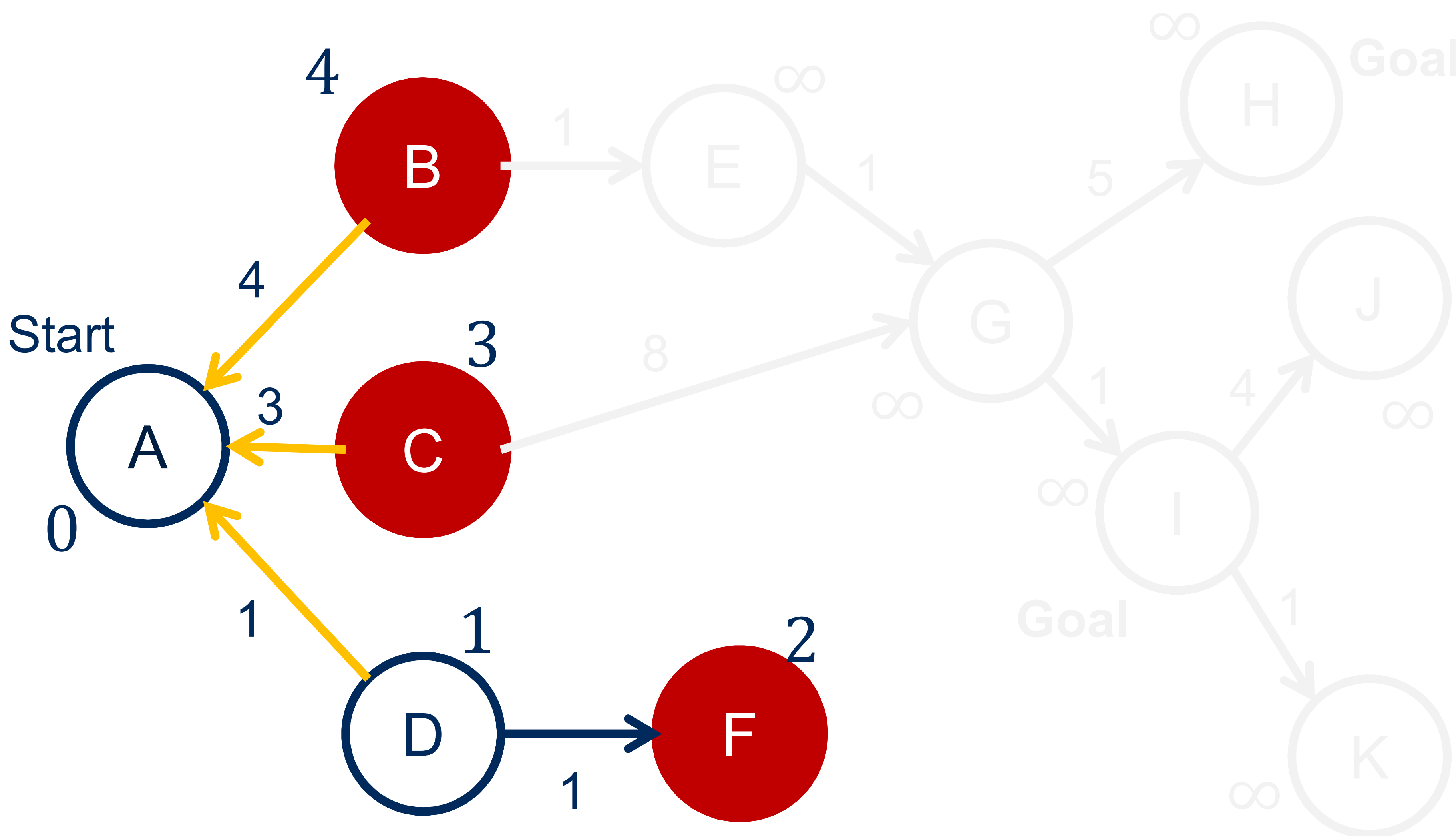

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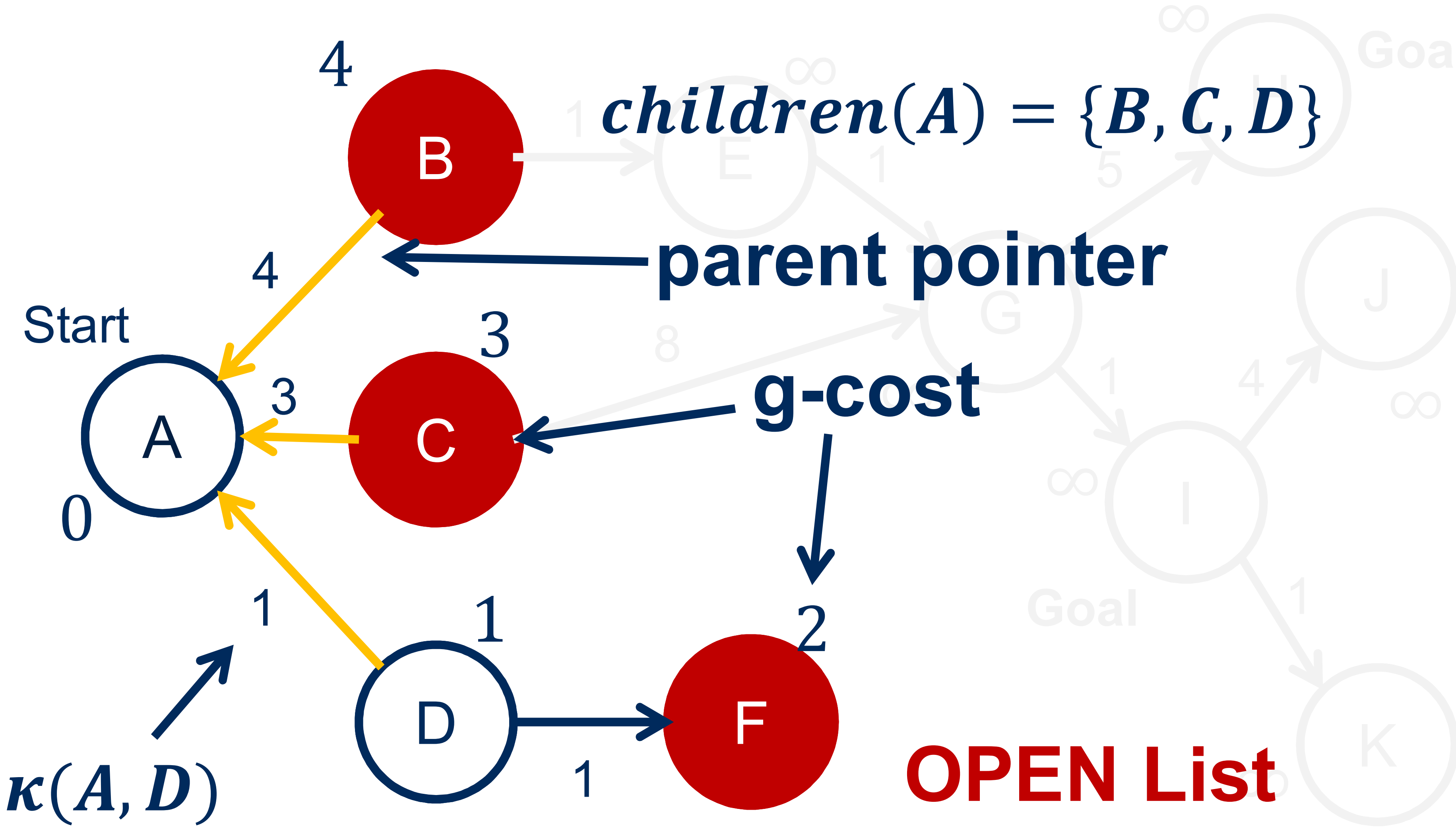

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```
def UniformCostSearch(s_I):
    OPEN ← {s_I}, CLOSED ← {},
    g(s_I) = 0, parent(s_I) = ∅
    while OPEN ≠ {}:
        p ← argmin_{s′∈OPEN} g(s′)
        if p is a goal, return path to p
        for c ∈ children(p):
            if c ∉ OPEN ∪ CLOSED:
                g(c) = g(p) + κ(p, c)
                parent(c) = p
                OPEN ← OPEN ∪ {c}
            else if g(c) > g(p) + κ(p, c):
                g(c) = g(p) + κ(p, c)
                parent(c) = p
                if c ∈ CLOSED:
                    OPEN ← OPEN ∪ {c}
                    CLOSED ← CLOSED − {c}
        OPEN ← OPEN − {p}, CLOSED ← CLOSED ∪ {p}
    return No solution exists
```

def UniformCostSearch(s_I):

$OPEN \leftarrow \{s_I\}, CLOSED \leftarrow \{\},$
$g(s_I) = 0, parent(s_I) = \emptyset$

while $OPEN \neq \{\}$:

p $\leftarrow$ $\text{argmin}_{\{s' \in OPEN\}} g(s')$

if p is a goal, **return** path to p

for $c \in children(p)$:

if $c \notin OPEN \cup CLOSED$:

$g(c) = g(p) + \kappa(p, c)$
$parent(c) = p$
$OPEN \leftarrow OPEN \cup \{c\}$

else if $g(c) > g(p) + \kappa(p, c)$:

$g(c) = g(p) + \kappa(p, c)$
$parent(c) = p$

if $c \in CLOSED$:

$OPEN \leftarrow OPEN \cup \{c\}$
$CLOSED \leftarrow CLOSED - \{c\}$

$OPEN \leftarrow OPEN - \{p\}, CLOSED \leftarrow CLOSED \cup \{p\}$

return No solution exists

Initialize Search

Get node from OPEN

```
def UniformCostSearch(s_I):
    OPEN ← {s_I}, CLOSED ← {},
    g(s_I) = 0, parent(s_I) = ∅
    while OPEN ≠ {}:
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                if c ∈ CLOSED:
                    OPEN ← OPEN ∪ {c}
                    CLOSED ← CLOSED − {c}
        OPEN ← OPEN − {p}, CLOSED ← CLOSED ∪ {p}
    return No solution exists
```

Generate and handle children

```
def UniformCostSearch(s_I):
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    g(s_I) = 0, parent(s_I) = ∅
    while OPEN ≠ {}:
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        if p is a goal, return path to p
        for c ∈ children(p):
            if c ∉ OPEN ∪ CLOSED:
                g(c) = g(p) + κ(p, c)
                parent(c) = p
                OPEN ← OPEN ∪ {c}
            else if g(c) > g(p) + κ(p, c):
                g(c) = g(p) + κ(p, c)
                parent(c) = p
                if c ∈ CLOSED:
                    OPEN ← OPEN ∪ {c}
                    CLOSED ← CLOSED − {c}
        OPEN ← OPEN − {p}, CLOSED ← CLOSED ∪ {p}
    return No solution exists
```

Generate and handle children

def UniformCostSearch(s_I):

$OPEN \leftarrow \{s_I\}, CLOSED \leftarrow \{\},$

$g(s_I) = 0, parent(s_I) = \emptyset$

while $OPEN \neq \{\}$:

p $\leftarrow \text{argmin}_{\{s' \in OPEN\}} g(s')$

if p is a goal, **return** path to p

for $c \in children(p)$:

if $c \notin OPEN \cup CLOSED$:

$g(c) = g(p) + \kappa(p, c)$

$parent(c) = p$

$OPEN \leftarrow OPEN \cup \{c\}$

else if $g(c) > g(p) + \kappa(p, c)$:

$g(c) = g(p) + \kappa(p, c)$

$parent(c) = p$

if $c \in CLOSED$:

$OPEN \leftarrow OPEN \cup \{c\}$

$CLOSED \leftarrow CLOSED - \{c\}$

$OPEN \leftarrow OPEN - \{p\}, CLOSED \leftarrow CLOSED \cup \{p\}$

return No solution exists

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$OPEN \leftarrow OPEN \cup \{c\}$
$CLOSED \leftarrow CLOSED - \{c\}$
$OPEN \leftarrow OPEN - \{p\}, CLOSED \leftarrow CLOSED \cup \{p\}$
return No solution exists

Close expanded node

def UniformCostSearch(s_I):

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$g(s_I) = 0, parent(s_I) = \emptyset$

while $OPEN \neq \{\}$:

p ← argmin$_{\{s' \in OPEN\}} g(s')$

if p is a goal, **return** path to p

for $c \in children(p)$:

if $c \notin OPEN \cup CLOSED$:

$g(c) = g(p) + \kappa(p, c)$
$parent(c) = p$

$OPEN \leftarrow OPEN \cup \{c\}$

else if $g(c) > g(p) + \kappa(p, c)$:

$g(c) = g(p) + \kappa(p, c)$
$parent(c) = p$

if $c \in CLOSED$:

$OPEN \leftarrow OPEN \cup \{c\}$

$CLOSED \leftarrow CLOSED - \{c\}$

CLOSED ← $CLOSED - \{p\}$

return No solution exists

Repeat

Uniform Cost Search

- UCS is completely **exhaustive** and **brute-force**
- Makes it prohibitively expensive

Depth	Nodes	Time	Memory
2	1100	.11 seconds	1 megabyte
4	111,100	11 seconds	106 megabytes
6	10^7	19 minutes	10 gigabytes
8	10^9	31 hours	1 terabytes
10	10^{11}	129 days	101 terabytes
12	10^{13}	35 years	10 petabytes
14	10^{15}	3,523 years	1 exabyte

Figure 3.11 Time and memory requirements for breadth-first search. The numbers shown assume branching factor $b = 10$; 10,000 nodes/second; 1000 bytes/node.

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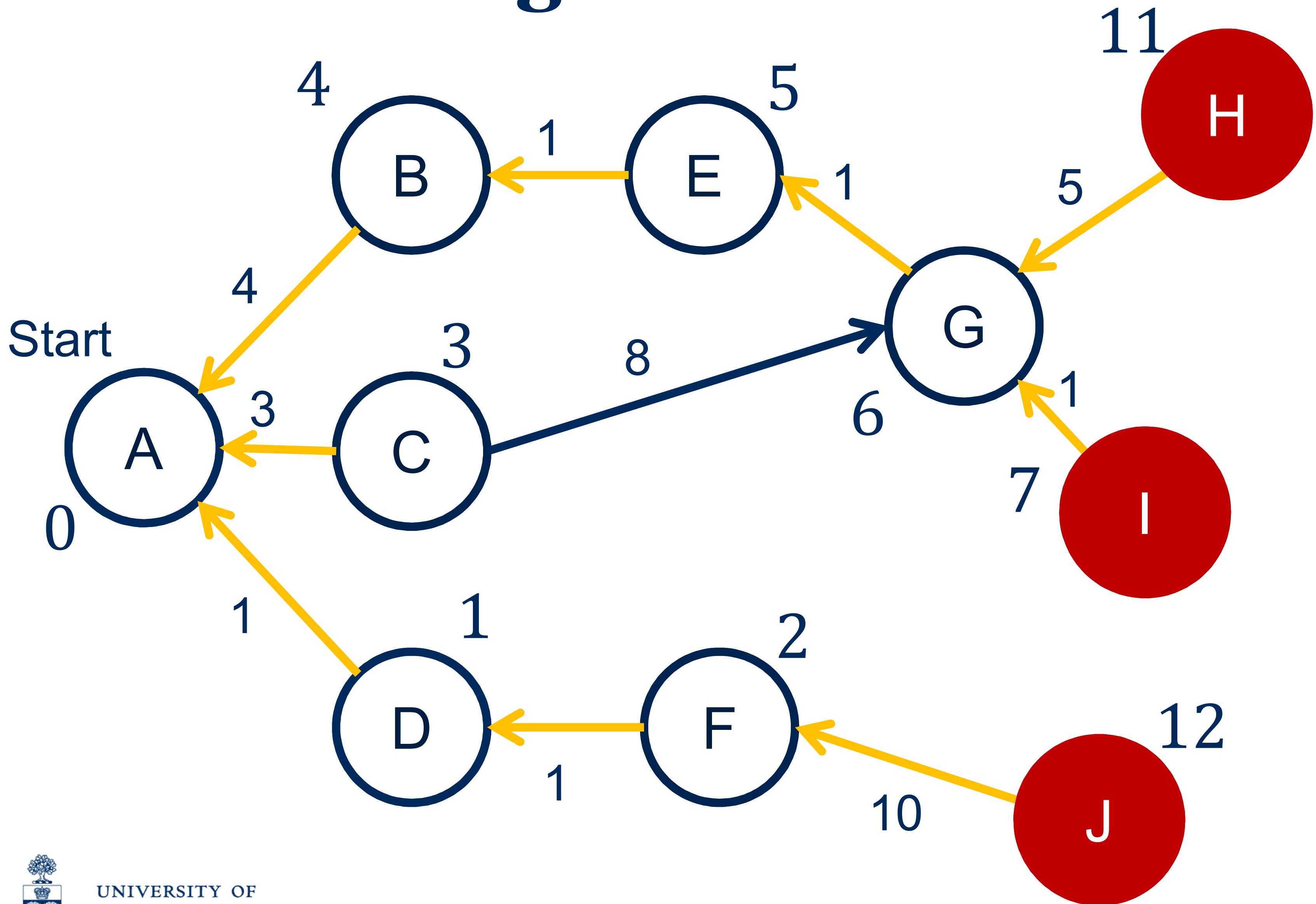
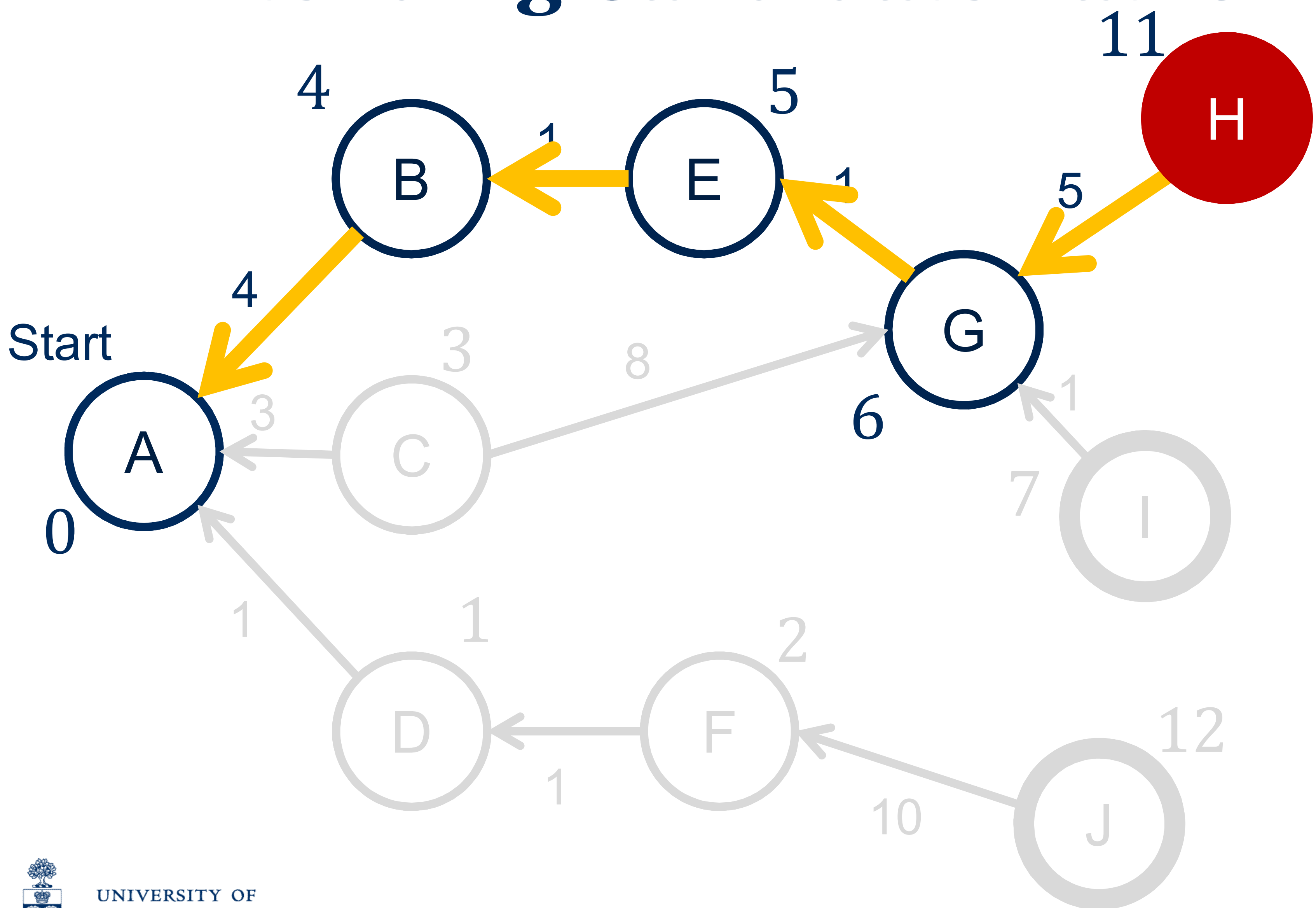
Extending Candidate Paths



Extending Candidate Paths



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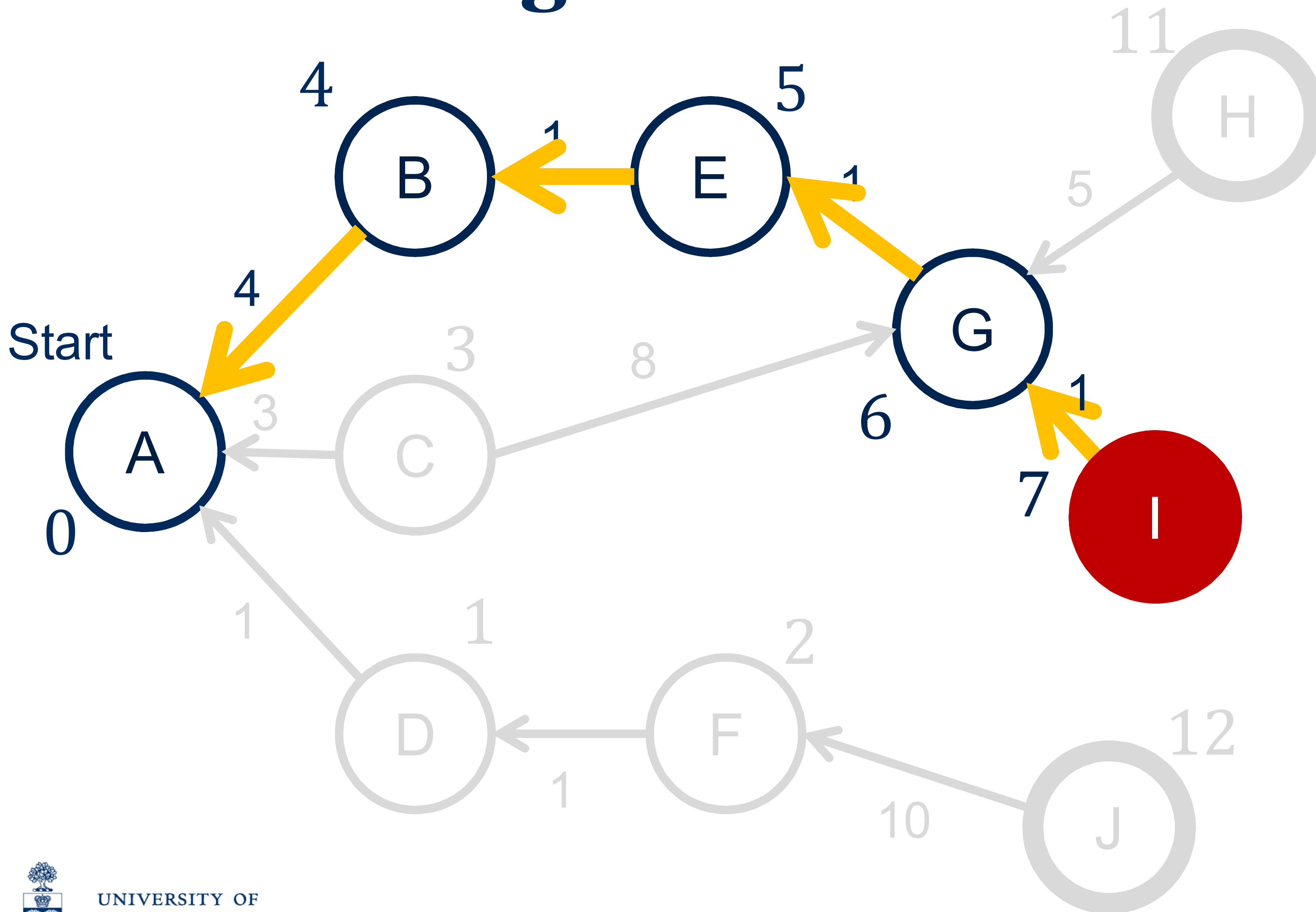
Extending Candidate Paths



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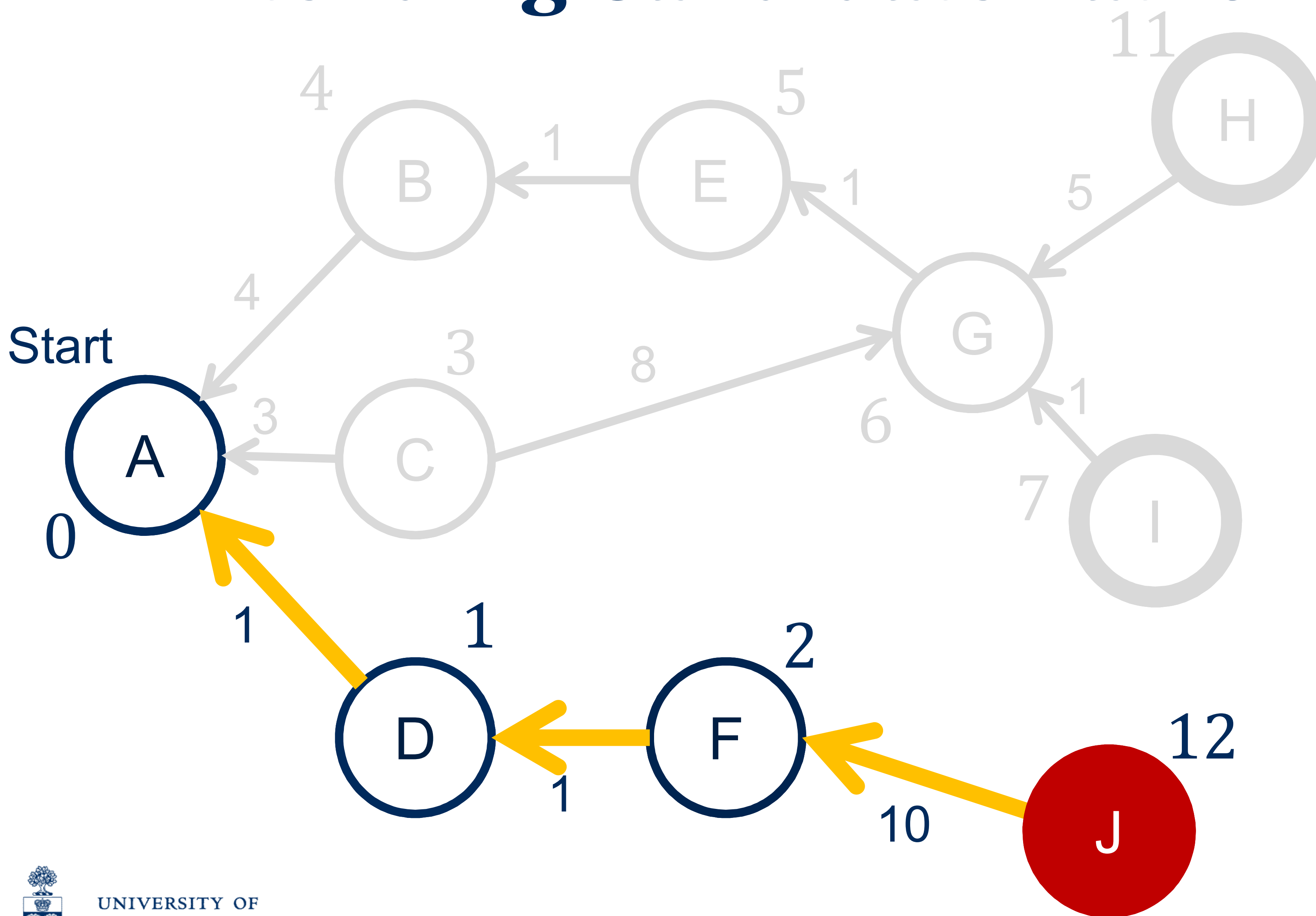
Extending Candidate Paths



Uniform Cost Search

- Iteratively extending some **candidate path**
- Uses the g-cost as the basis of this selection
  - Only info that uniform cost search has about a state
  - Only "uses" the transition function
- But each vertex represents a state
  - There is more information that can be used

Uniform Cost Search

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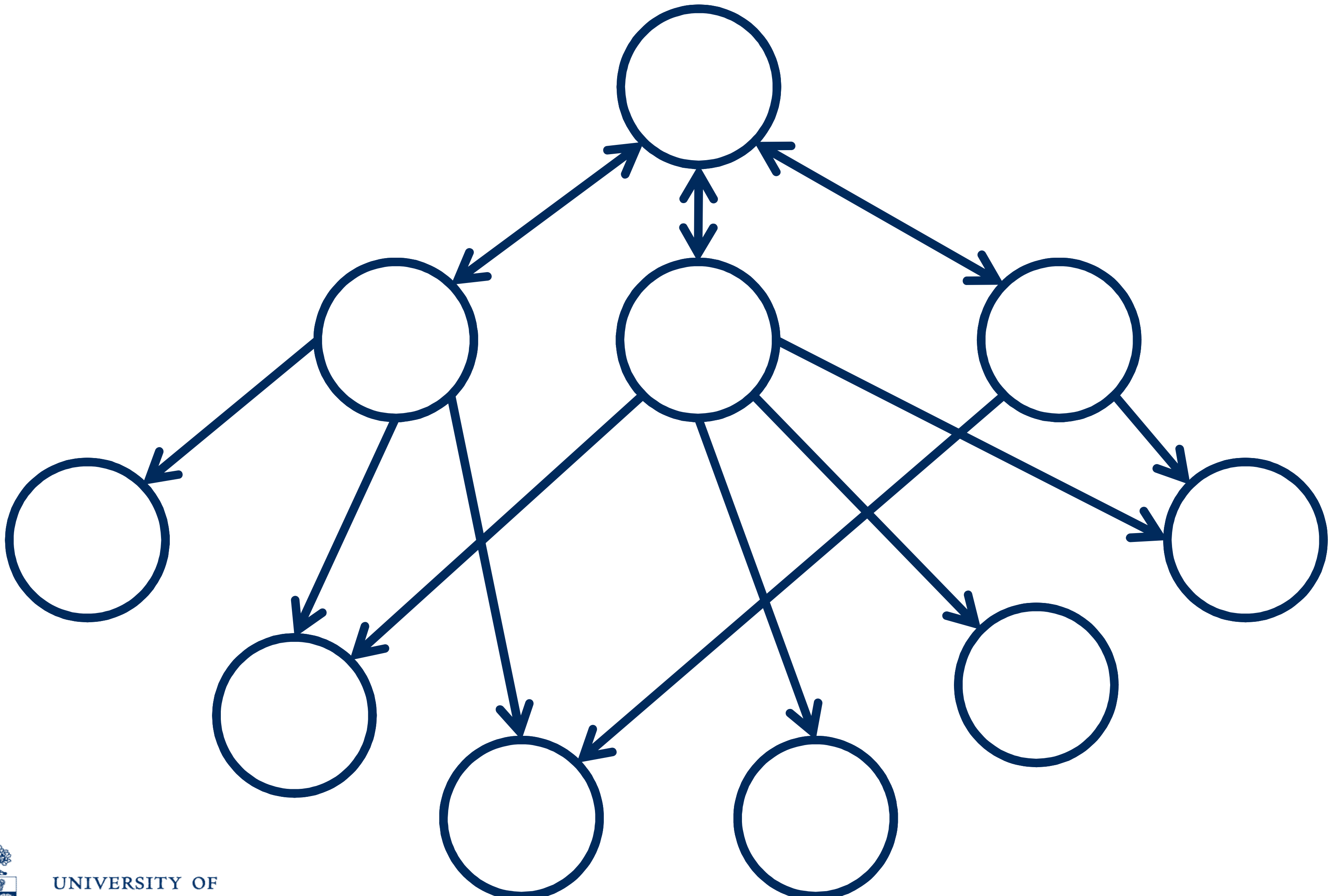
Graph Underlying Floortile



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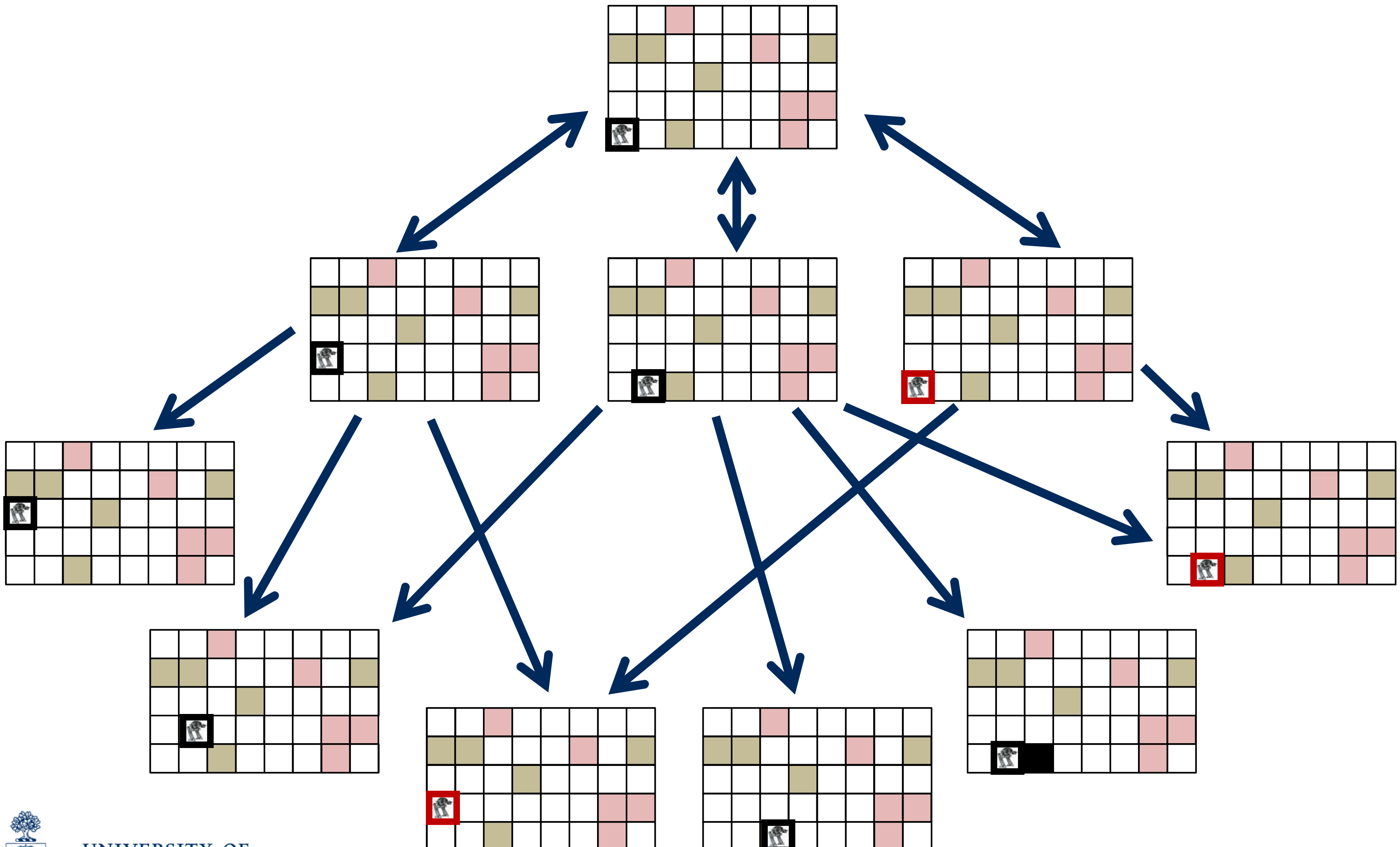
States Corresponding to Vertices



Heuristic Guidance

- A **heuristic function** *h* is a function from states* to the non-negative real values
  - Estimate the cost to reach the goal from the state
  - Other algorithms use such functions to change how they determine the order for extending candidate paths
- Often based on domain knowledge or domain simplification

* Or sometimes candidate paths to real values

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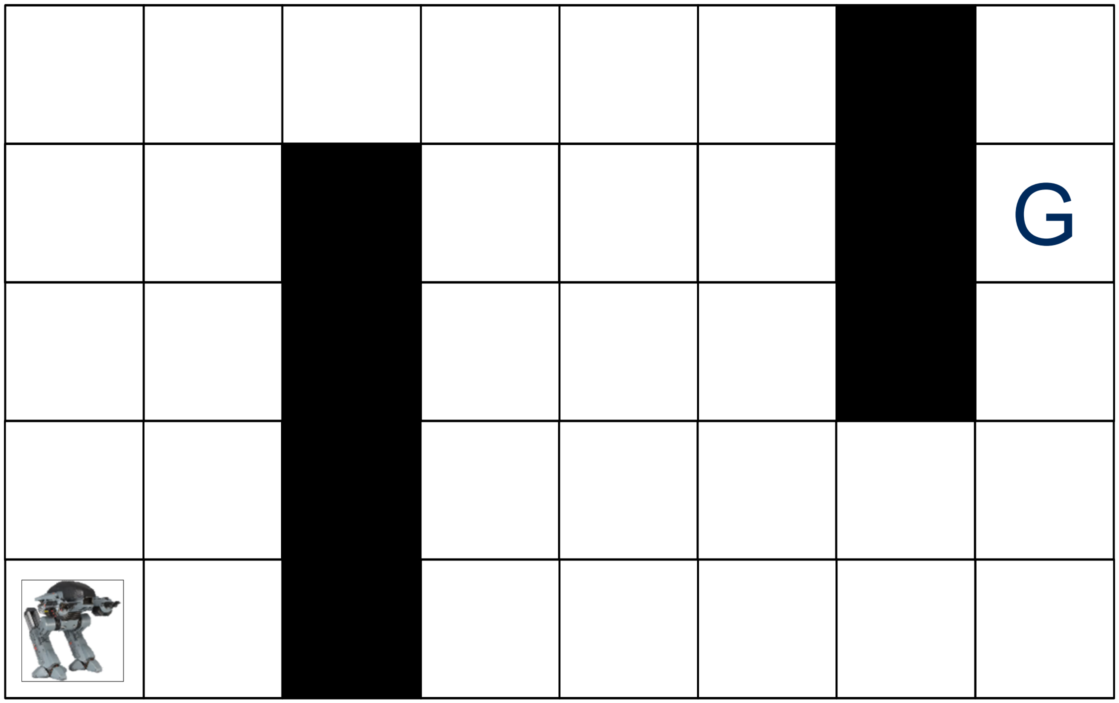
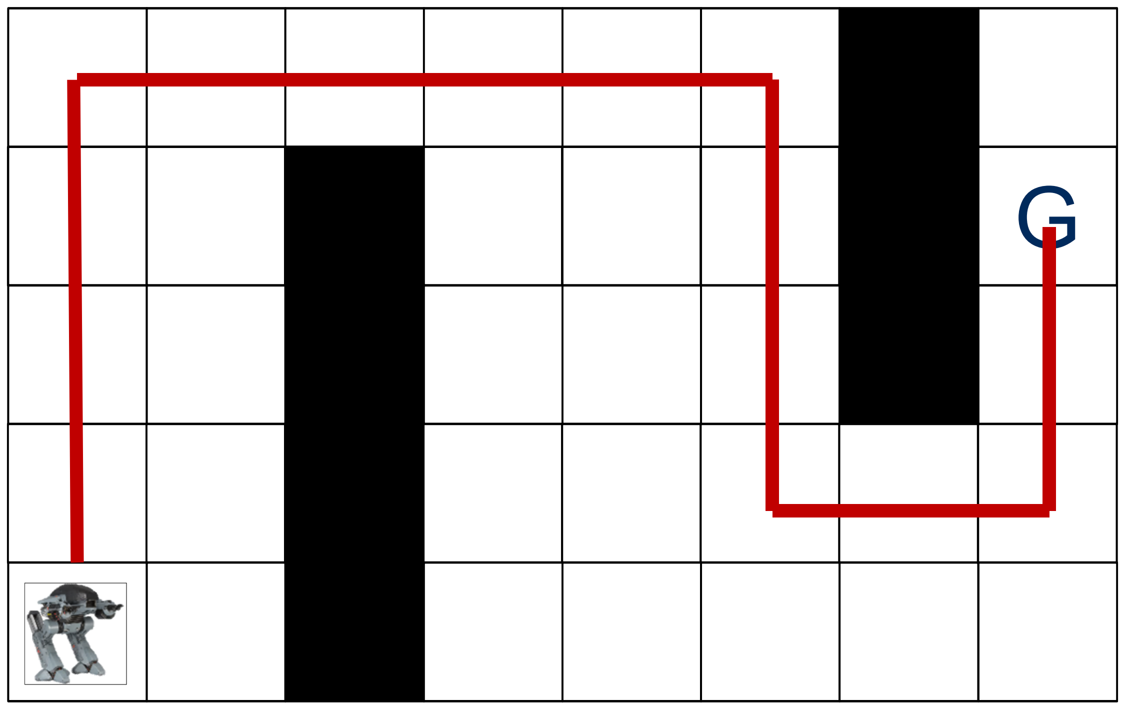
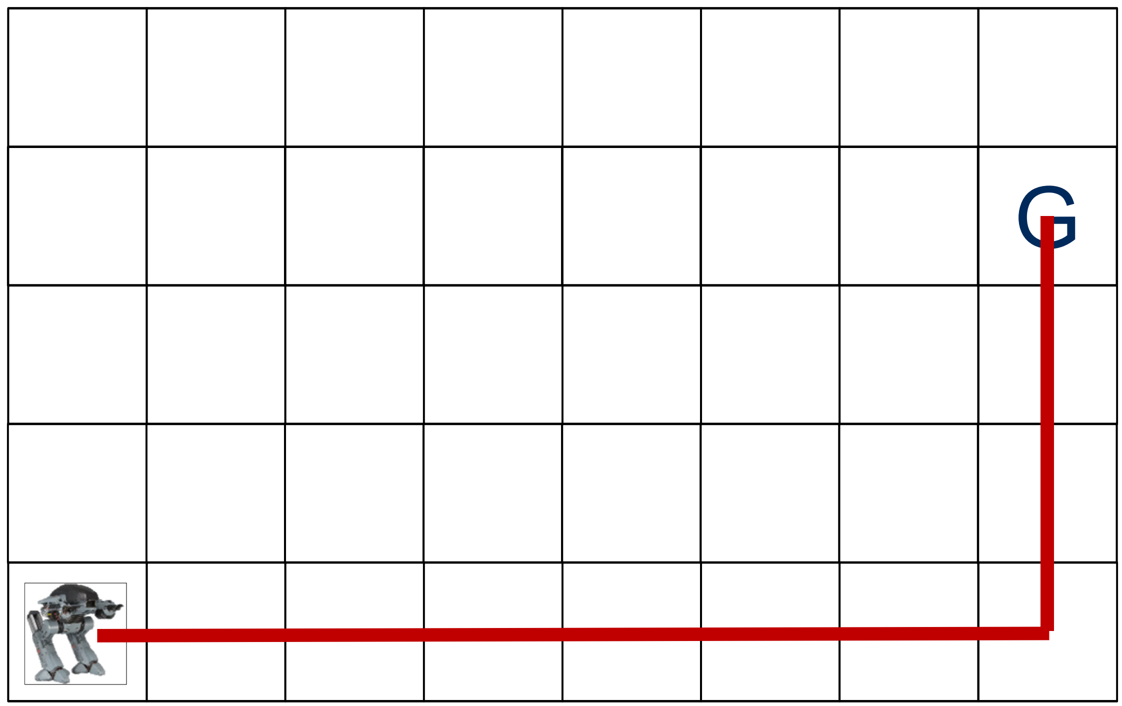
Pathfinding



Pathfinding



Pathfinding



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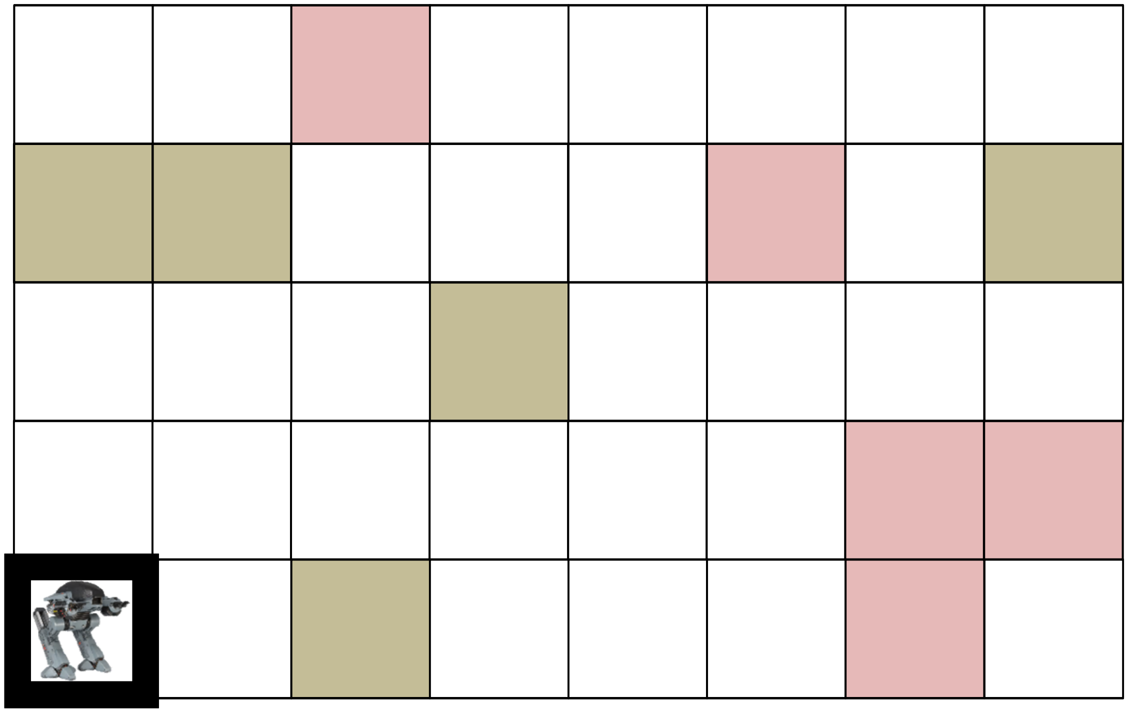
Floortile from IPC 2011



- What are possible heuristics or simplications here?

Automatic Heuristic Generation

- Can use domain knowledge

- Many automatic heuristic generation techniques
  - Delete relaxation
  - Pattern databases
  - Landmark-based heuristics
  - Merge-and-Shrink
  - Counterexample guided abstraction refinement heuristics
  - …

Automatic Heuristic Generation

- Can use domain knowledge

- Many automatic heuristic generation techniques
  - **Delete relaxation**
  - Pattern databases
  - Landmark-based heuristics
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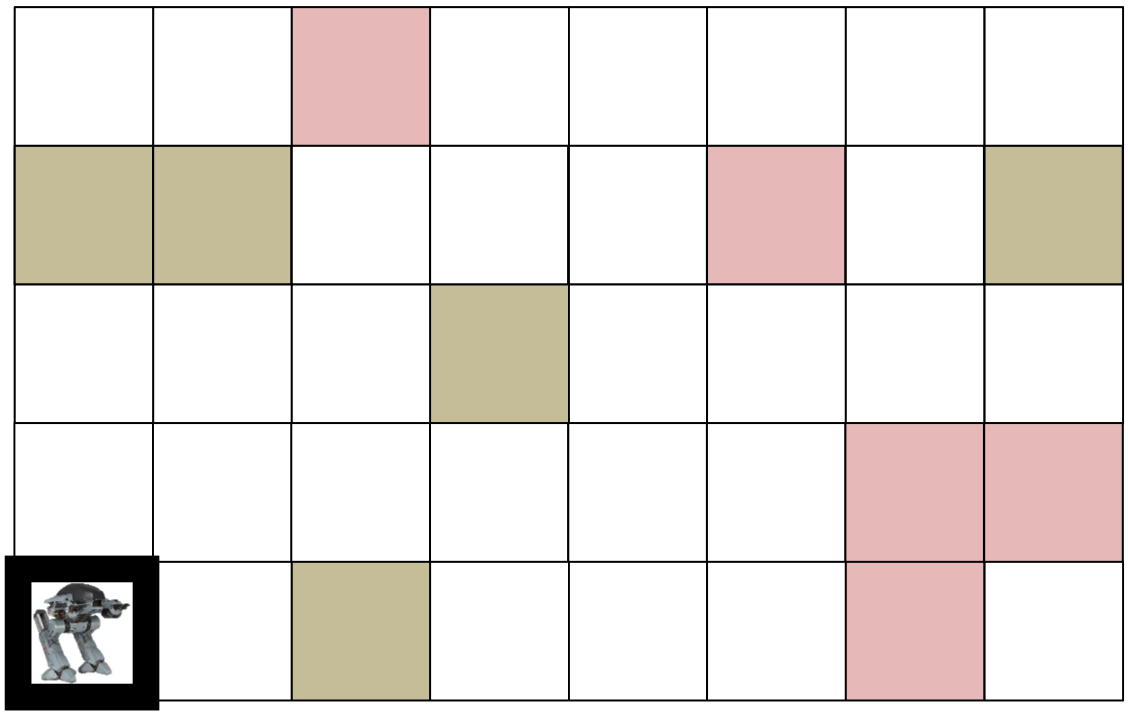
  - …

Delete Relaxation

- Can only achieve new facts, never delete them



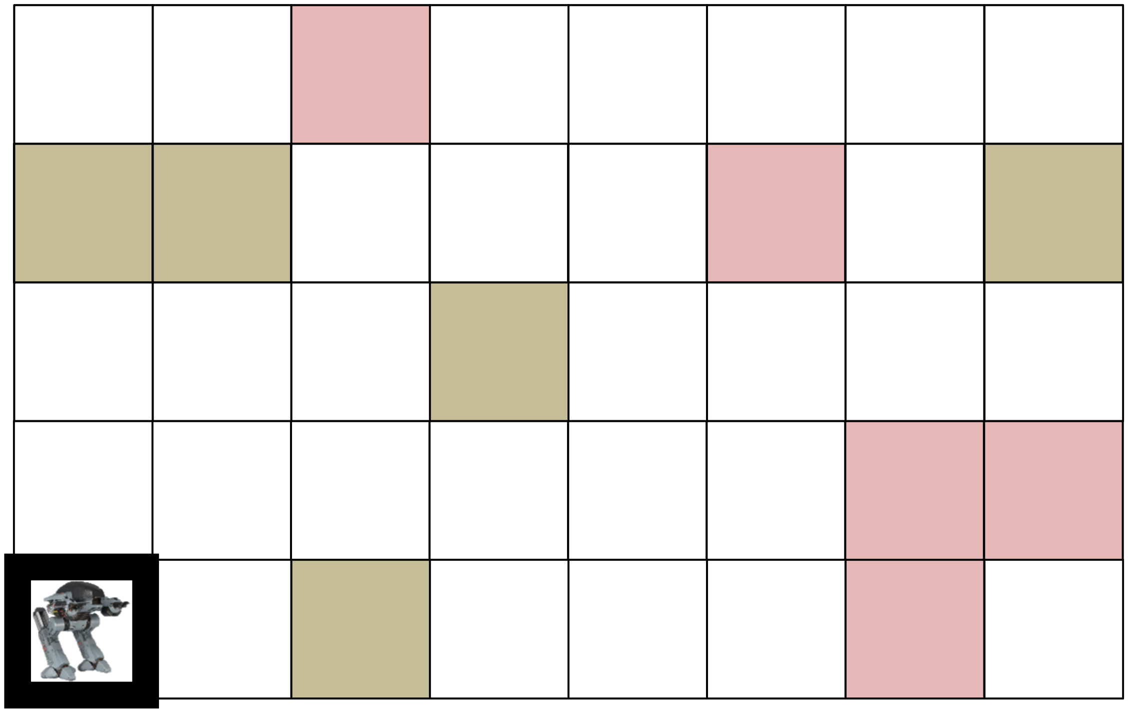
Move Action: MOVE-0-0-0-1

Pre: AT-0-0, WHITE-0-1

Post: AT-0-1, not(AT-0-0)

Delete Relaxation

- Can only achieve new facts, never delete them



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Post: AT-0-1, ~~not(AT-0-0)~~

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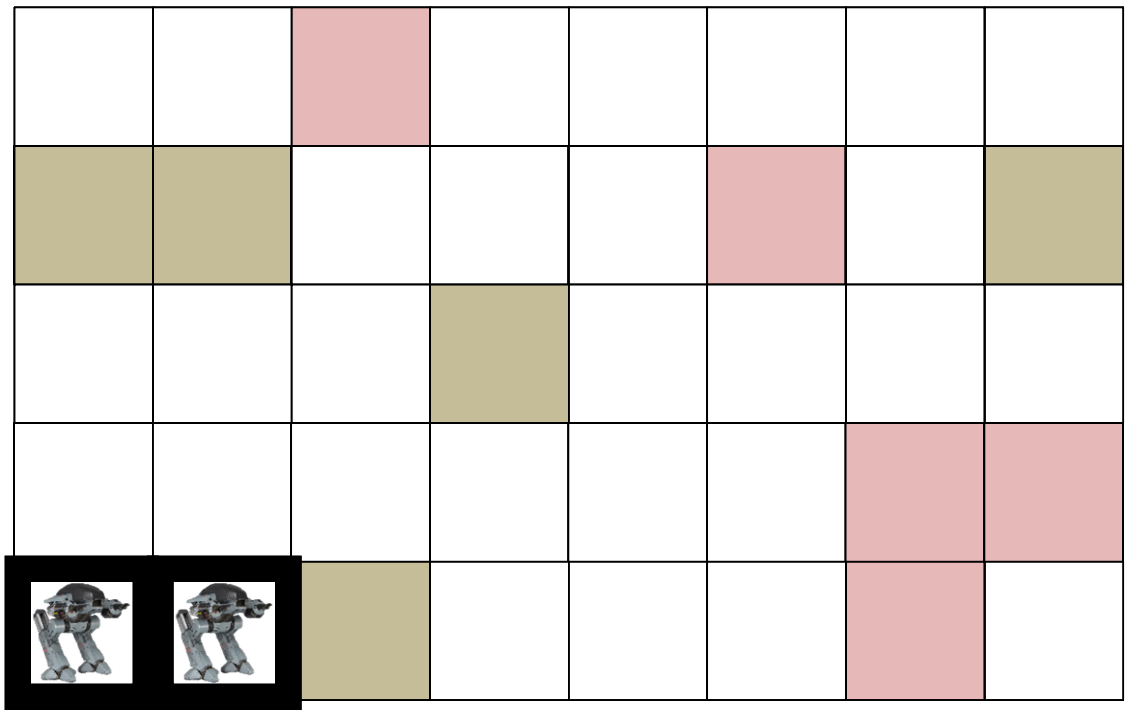

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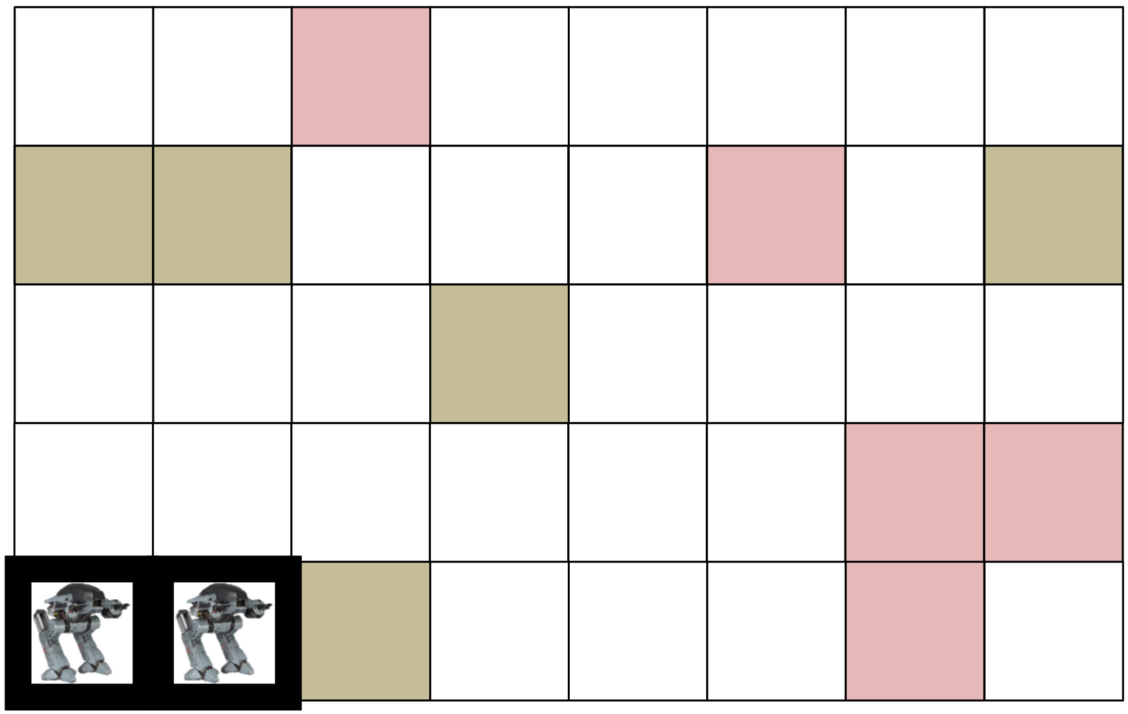

Paint Action: PAINT-B-0-1-0-2

Pre: AT-0-1, LOADED-B,
WHITE-0-2, NEED-B-0-2

Post: BLACK-0-2, not(WHITE-0-2)

Delete Relaxation

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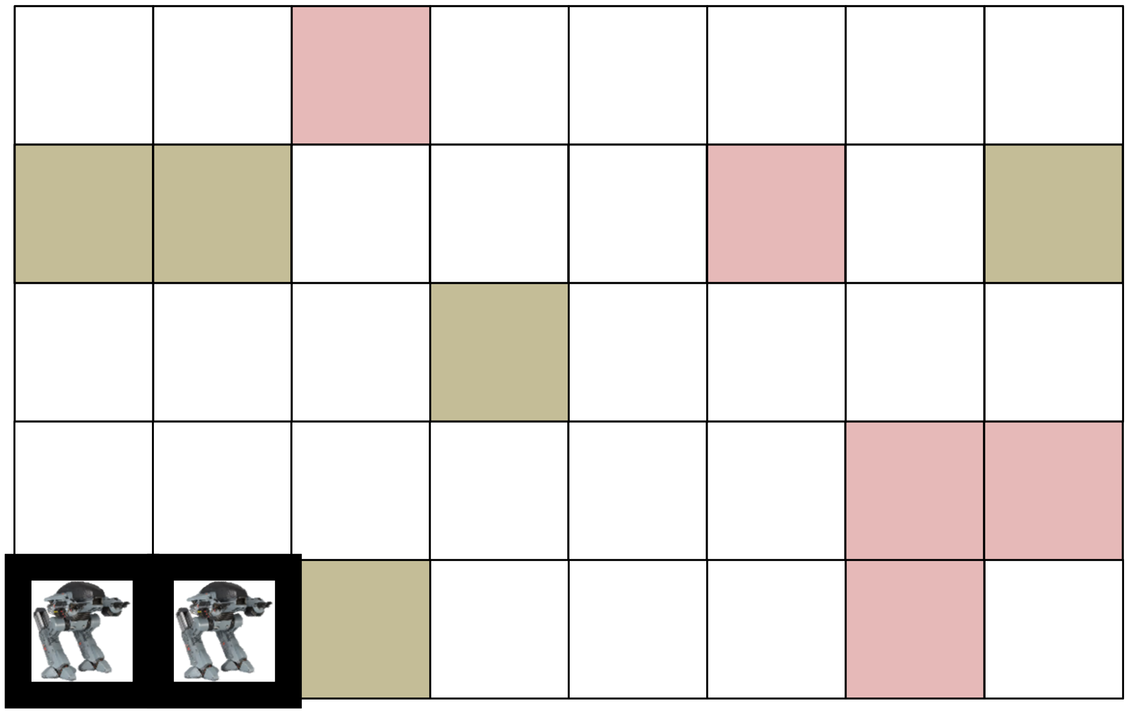

Paint Action: PAINT-B-0-1-0-2

Pre: AT-0-1, LOADED-B,
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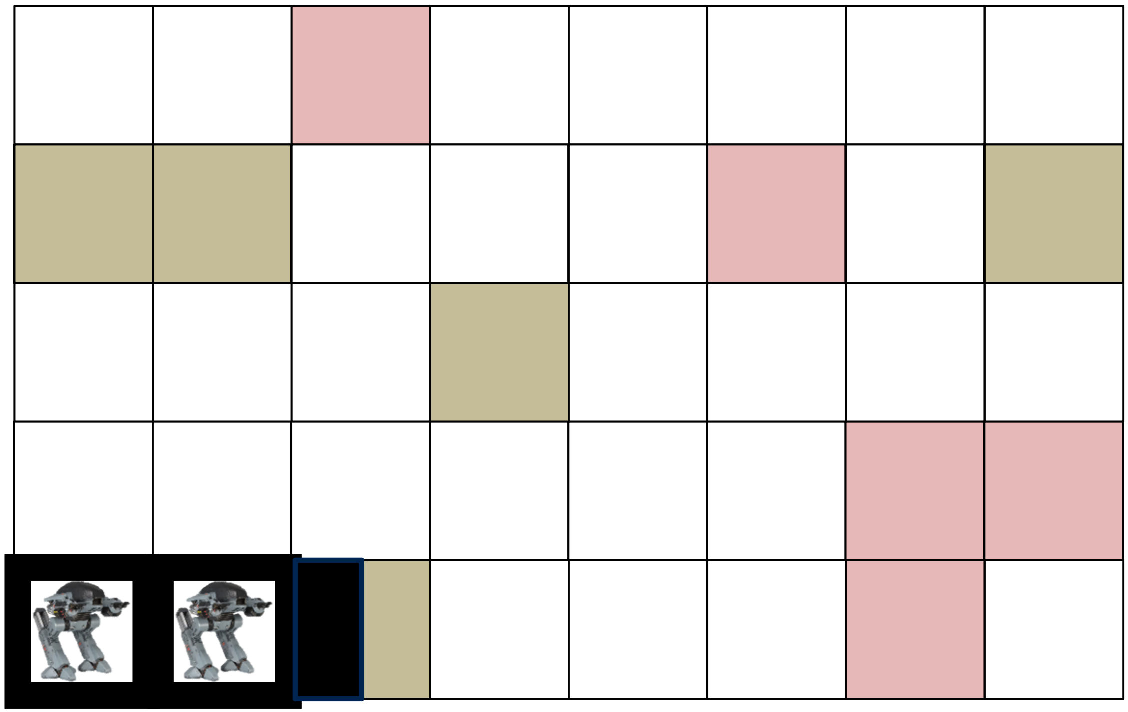

Paint Action: PAINT-B-0-1-0-2

Pre: AT-0-1, LOADED-B,
WHITE-0-2, NEED-B-0-2

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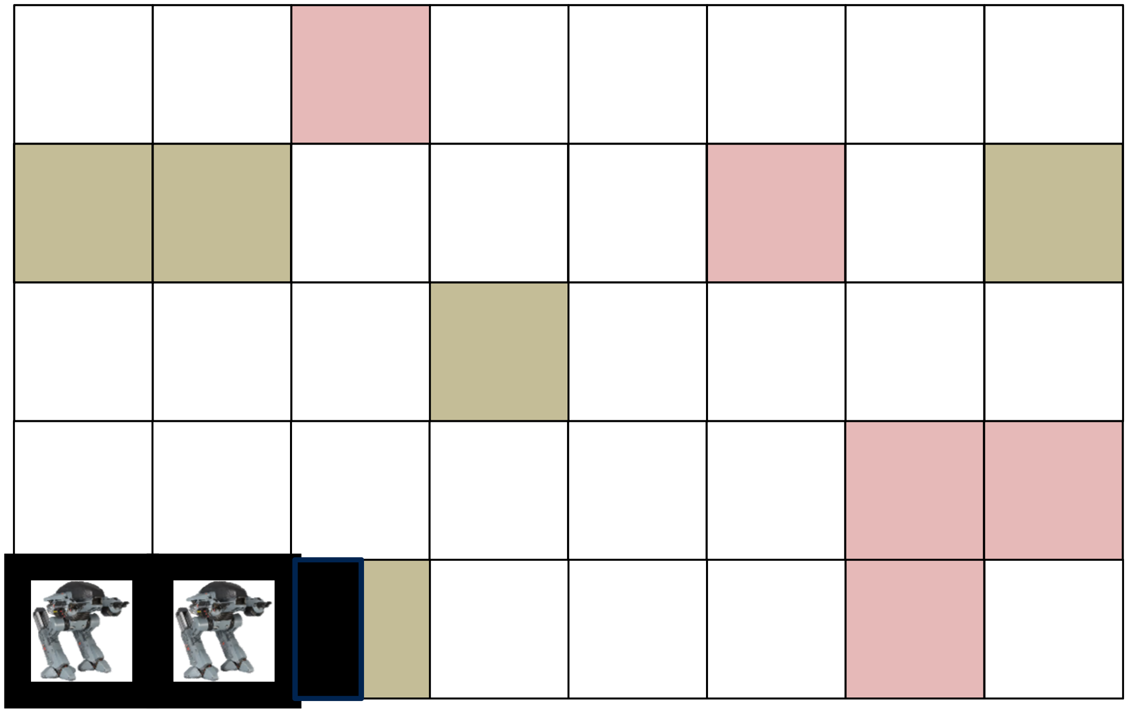

Load Action: LOAD-RED

Pre: LOADED-B

Post: LOADED-R, not(LOADED-B)

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Delete Relaxation

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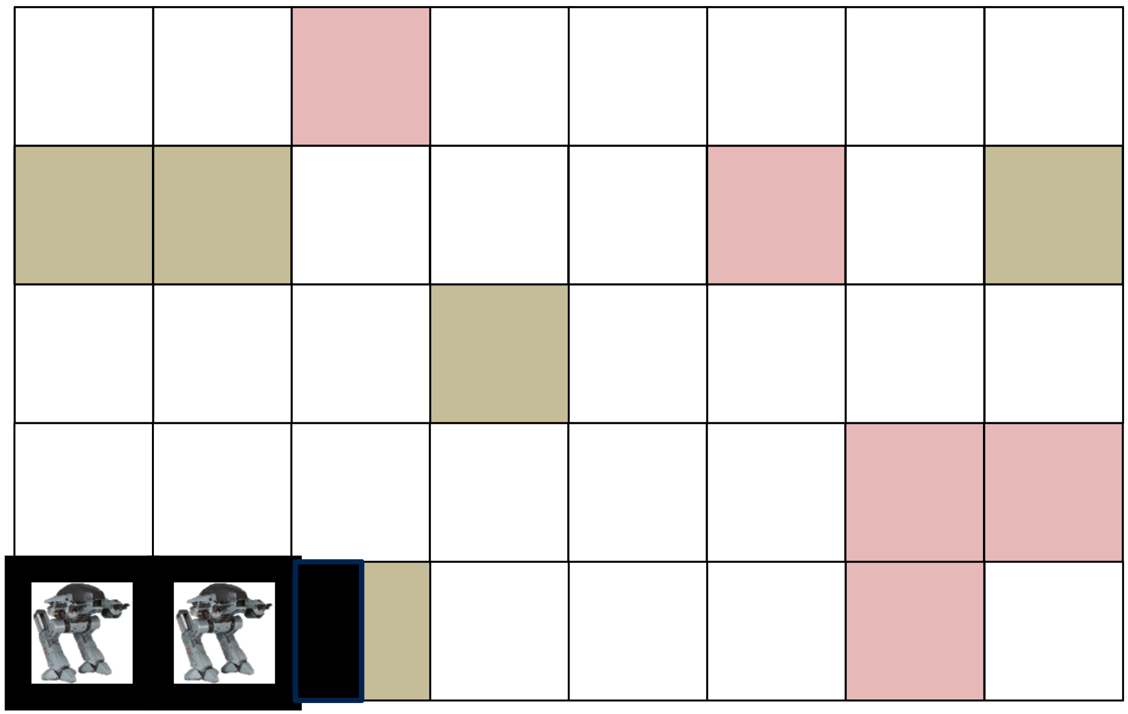

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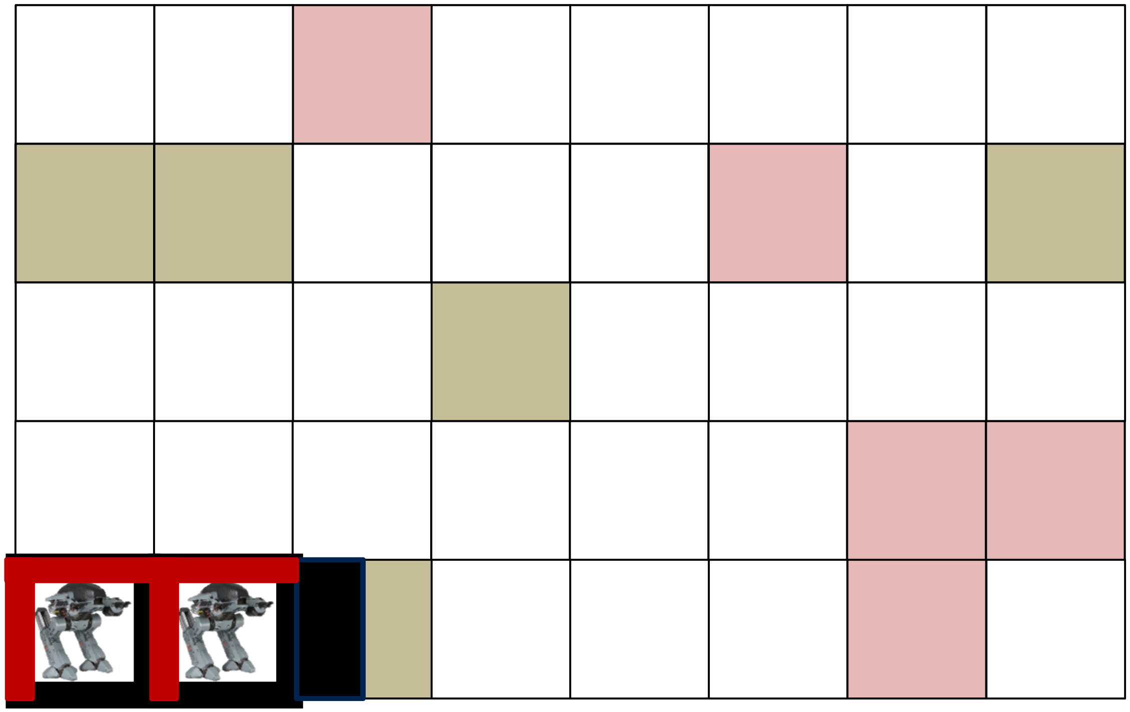

Load Action: LOAD-RED

Pre: LOADED-B

Post: LOADED-R, ~~not(LOADED-B)~~

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Delete Relaxation

- Still NP-complete to optimally solve delete relaxed problems
  - Better than PSPACE-hard, but still …

- Do have polynomial ways to solve them suboptimally or come up with a lower bound

Summary

- Can solve planning using graph search
  - Generate graph and use Dijkstra's search
- Can incrementally generate the graph and stop early
  - Uniform cost search is this adjustment
- Uniform cost search is only using transition function
  - Ignoring state information
- Heuristic functions use state information to generate an estimate of the cost to a goal