Let the alphabet be $\Sigma = \{a, b\}$

1. Are regular expressions $(a+b)^*$ and $a^*+b^*$ equivalent? Explain.
   Solution: Let $R_1 = (a+b)^*$ and $R_2 = a^*+b^*$. $R_1 \neq R_2$, because $R_1$ includes all strings in the alphabet $\Sigma$ while $R_2$ includes repetitions of $a$ (including 0 repetitions) or repetitions of $b$, but no strings that include both $a$ and $b$. The corresponding NFA are different as well:

   ![NFA Diagram]

2. Draw a DFA corresponding to the regular expression $(a+b)(a+b)^*(a^*+b^*)$. Write down the corresponding state invariant that you could use to prove the equality of your DFA to the regular language represented by the provided regexp. You don’t need to provide the proof.
   Solution: First, note that $L(a^*+b^*) \subseteq L(a+b)^*$ and not only that, but any string that can be generated by $(a+b)^*(a^*+b^*)$ can be generated by $(a+b)^*$ due to the definition of the Kleene’s star. Hence, we can simplify $(a+b)(a+b)^*(a^*+b^*) = (a+b)(a+b)^*$. The corresponding DFA is

   ![DFA Diagram]

   State invariants are then as follows:

   \[ \delta'(q_0, s) = \begin{cases} 
   q_1 & \text{if } s \text{ starts with an } a \text{ or } b \\
   q_0 & \text{otherwise} 
   \end{cases} \]
3. Consider an FSA $M_1$:

(a) Is this a DFA or an NFA? Why?
Answer: This is an NFA, because $\delta(q_1, b) = \{q_1, q_0\}$, i.e. there is more than one path to take upon observing $b$ in state $q_1$. Also, the transition $\delta(q_0, b)$ is not determined but sometimes dead states are omitted even in drawing DFSA, so you have to be careful calling FSA an NFA just because dead states are missing.

(b) Write down the language $L$ that it represents (a sentence describing all strings included in the language $L$)
Answer: This language contains all strings that start with an $a$.

(c) Write down the complement $\overline{L}$ of $L$, i.e. $\overline{L} = \Sigma^* - L$ in one sentence
Answer: Language $\overline{L}$ contains all strings in $\Sigma^*$ that do not start with an $a$ (including empty string).

(d) Draw an FSA for $\overline{L}$

4. Consider a regexp $R_1$: $a(ba^*)(a^* + b^*)$

(a) Draw an NFA $M_2$ corresponding to the $R_1$ above

Solution: There are many answers to this question, including

The second solution follows from $a(ba^*)(a^* + b^*) = ab(a^* a^* + a^* b^*) = ab(a^* + a^* b^*) = aba^* b^*$

(b) Write down the language $L$ that it represents (a sentence describing all strings)
Answer: $L$ contains all strings that start with $ab$, concatenated with repetitions of $a$ (including zero repetitions of $a$) followed by repetitions of $b$ (including zero repetitions of $b$).

(c) Draw a corresponding DFA