Outline

Regular Expressions

NFAs

Notes
Another way to define languages

In addition to the language accepted by DFSA $L(M)$ and set description $L = \{ \ldots \}$.

Definition: The regular expressions (regexps or REs) over alphabet $\Sigma$ is the smallest set such that:

1. $\{\}, \epsilon$ are REs
2. $a$ for every $a \in \Sigma$ are REs over $\Sigma$
3. if $R$ and $S$ are REs over $\Sigma$, then so are:
   - $R + S$ (union) — lowest precedence operator
   - $RS$ (concatenation) — middle precedence operator
   - $R^*$ (star) — highest precedence
Regular Expression to Language

- $L(\emptyset) = \emptyset$ (the empty language — no strings!)
- $L(\epsilon) = \{\epsilon\}$ (the language consisting of just the empty string)
- $L(x) = \{x\}$ (the language consisting of the one-symbol string)

- $L(S + T) = L(S) \cup L(T)$
- $L(ST) = L(S)L(T)$
- $L(T^*) = L(T)^*$
RE Examples

- $L(a + b) = \{a, b\}$
- $L(ab) = \{ab\}$
- $L((a + b)a) = \{aa, ba\} = L(aa + ba)$
- $L(a^*) = \{\epsilon, a, aa, aaaa, \ldots\}$
- $L(aa^*) = \ldots$
- $L((ab)^*) = \ldots$
- $L(a^*b^*) = \ldots$
RE Examples

- \( L((a + b)^*) = \)

- \( L(a^* + b^*) = \)

- \( L((a + b)(a + b)^*) = \)

- All strings of \( a \)'s and \( b \)'s that have the same first and last symbol?

- All strings of \( a \)'s and \( b \)'s that contain at least 1 \( a \)?
Proving a RE defines a specified language

$L = \{ x \in \{0,1\}^* \mid x \text{ begins and ends with a different bit} \}$
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RE Identities

- communitativity of union: \( R + S \equiv S + R \)
- associativity of union: \((R + S) + T \equiv R + (S + T)\)
- associativity of concatenation: \((RS)T \equiv R(ST)\)
- left distributivity: \(R(S + T) \equiv RS + RT\)
- right distributivity: \((S + T)R \equiv SR + TR\)
- identity for union: \(R + \emptyset \equiv R\)
- identity for concatenation: \(R\varepsilon \equiv R \equiv \varepsilon R\)
- annihilator for concatenation: \(\emptyset R \equiv \emptyset \equiv R\emptyset\)
- idempotence of Kleene star: \((R^*)^* \equiv R^*\)
Another RE example...

... and reminder about proving equality of sets
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Non-deterministic Finite Automata
NFA Example
NFA that accepts $L((010 + 01)^*)$
NFA Example

\[ L = \{ w \in \{0\}^* : |w| \text{ is a multiple of 2 or 3} \} \]
NFAs vs DFAs vs REs

What can an NFA do that a DFA can’t?
Properties of Regular Languages

- Regular languages are *closed* under certain operations

- Applying these operations to regular languages results in regular languages
Closure Properties of Regular Languages
Closure Properties of Regular Languages
Define a DFA/NFA/RE to accept $L = \{0^n1^n : n \geq 0\}$
Proving non-regularity
Proving non-regularity
Proving non-regularity