(PRINT) Name: $\qquad$
$\qquad$

Signature: $\qquad$ Tutorial room $\qquad$ Total Mark: $\qquad$ /25

## CSC 200—Social and Economic Networks

Quiz 4, December 3, 2015
You are allowed to use one, two-sided $8.5 "$ by $11 "$ sheet of handwritten notes. No other materials or aids of any type are permitted. You may use other side of quiz paper for your answers to the quiz question.

Time: 25 minutes; Total Marks: 25

Consider a matching market in which there are four houses, $\mathrm{H} 1, \mathrm{H} 2, \mathrm{H} 3$, and H 4 , for sale in a certain neighborhood. There are four buyers $\mathrm{W}, \mathrm{X}, \mathrm{Y}$, and Z wishing to purchase these houses. The buyers have the following valuations (repeated on other side of paper):

|  | H 1 | H 2 | H 3 | H 4 |
| :---: | :---: | :---: | :---: | :---: |
| W | 2 | 7 | 5 | 4 |
| X | 6 | 8 | 5 | 4 |
| Y | 3 | 2 | 5 | 4 |
| Z | 4 | 7 | 2 | 1 |

Suppose we run the ascending price "auction" mechanism described in Ch. 10 (and Lectures 18 and 19 to determine market clearing prices), but without using the price reduction step (i.e., we do not insist that the smallest price is 0 ). Recall that we start with all house prices set to $\$ 0$ and then at each round of the auction, if there is a constricted set of buyers in the preferred seller graph, then we trigger price increases (of $1 \$$ ) for one or more houses. Run this auction using the following rule: if there is more than one possible constricted set of buyers at any round of the auction, choose a minimal set of constricted buyers to determine which house prices increase. For each round of the auction, show each of the following: i. prices for each house; ii. the preferred seller graph; iii. if the prices are not market clearing at that round, the minimal constricted set of buyers you selected and thus the houses whose prices will increase; and iv. if the prices are market clearing at that round, a perfect matching that assigns a house to each buyer.

|  | H1 | H2 | H3 | H4 |
| :---: | :---: | :---: | :---: | :---: |
| W | 2 | 7 | 5 | 4 |
| X | 6 | 8 | 5 | 4 |
| Y | 3 | 2 | 5 | 4 |
| Z | 4 | 7 | 2 | 1 |
| SOLUTION |  |  |  |  |

1. At start of round $1, \mathrm{H} 2$ is a minimal constrictred set demanded by $\mathrm{W}, \mathrm{X}, \mathrm{Z}$ so we increase the price of H 2 to be $\$ 1$.
2. At start of round $2, \mathrm{H} 2$ is still a minimal constricted set demanded by X and Z so price of H 2 is now \$2
3. At start of round 3, H2 and H 3 are a minimal constricted set demanded by $\mathrm{W}, \mathrm{Y}, \mathrm{Z}$ so price of H 2 becomes $\$ 3$ and price of H 3 becomes $\$ 1$.
4. At start of round 4, there are no constricted sets and the perfect matchings are (H1,X), (H2,Z), (H3,W) $(\mathrm{H} 4, \mathrm{Y})$ or $(\mathrm{H} 1, \mathrm{X}),(\mathrm{H} 2, \mathrm{Z}),(\mathrm{H} 3, \mathrm{Y})(\mathrm{H} 4, \mathrm{~W})$.
